A Model for the Binary Asteroid 2017 YE5

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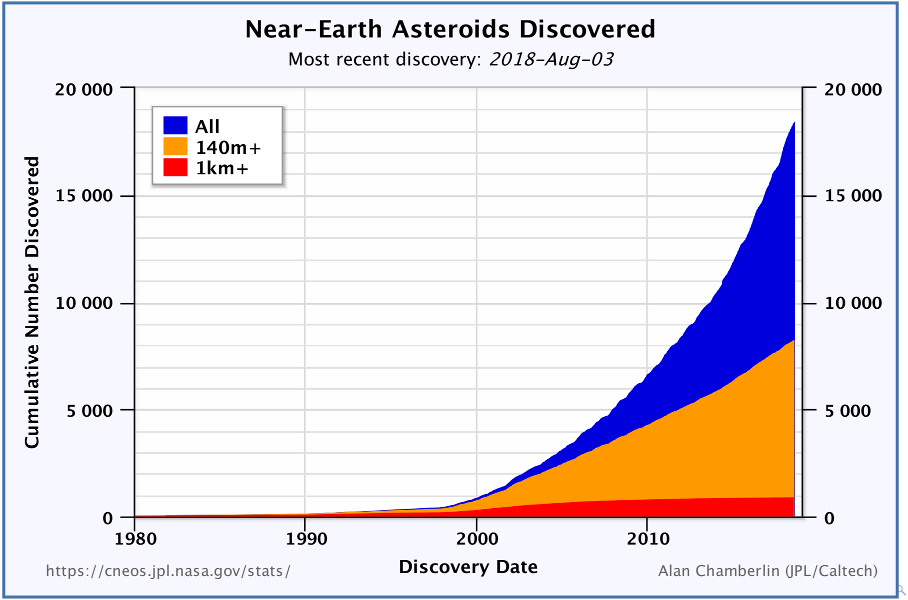
December 8, 2018

A more accurate title might be

A Four-Body Problem inspired by the Binary Asteroid 2017 YE5

Near Earth Asteroids

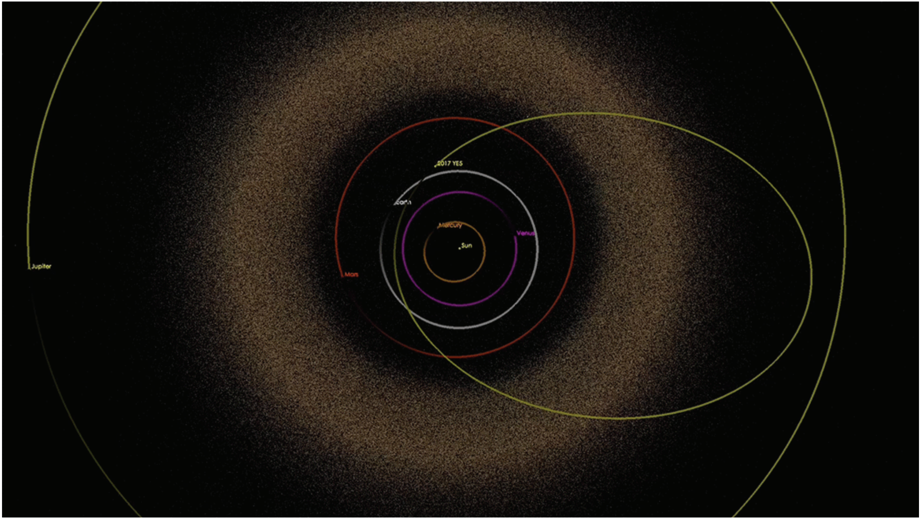


2017 YE5

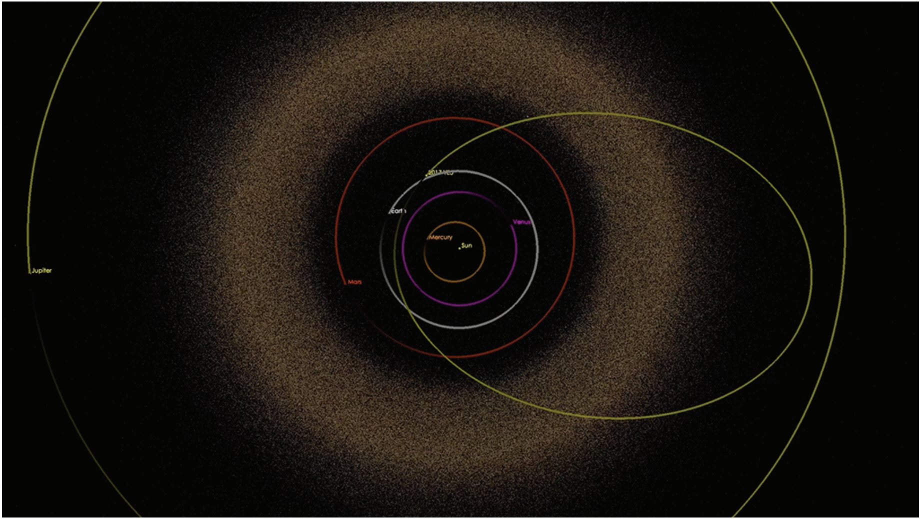
Discovered by C. Rinner, M. Ory, and B. Zouhair in December 2017.
Orbital Characteristics

- Aphelion 4.82 AU
- Perihelion 0.8171 AU
- Semi-Major Axis 2.82 AU
- Eccentricity 0.712
- Orbital period 4.74 yr
- Mean anomoly 349.0°
- Mean motion 0.2081° per day
- Inclination 6.21°
- Longitude of ascending node 103.96°
- Argument of perihelion 110.77°

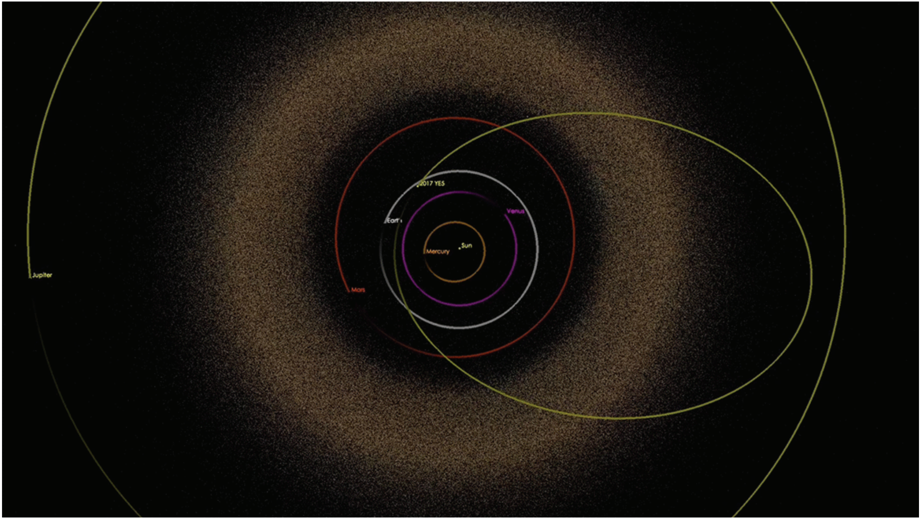
Animation of 2017 YE5 (1)



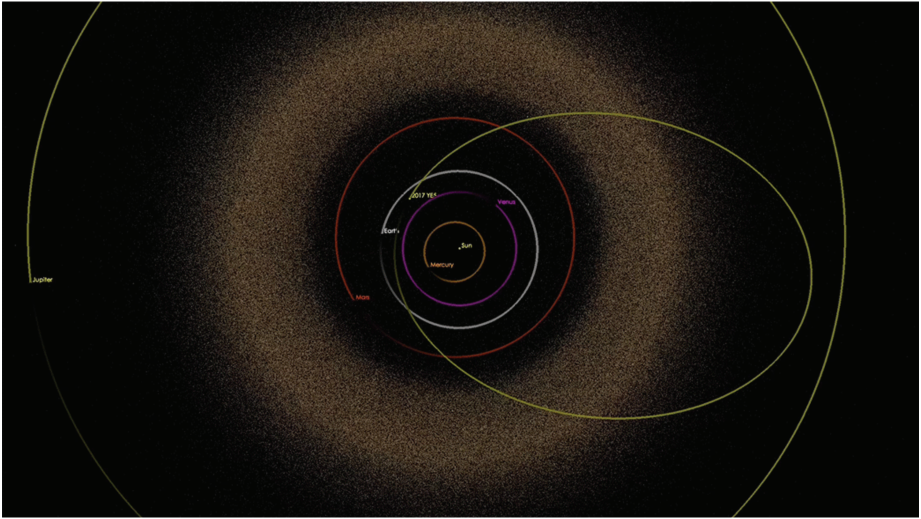
Animation of 2017 YE5 (2)



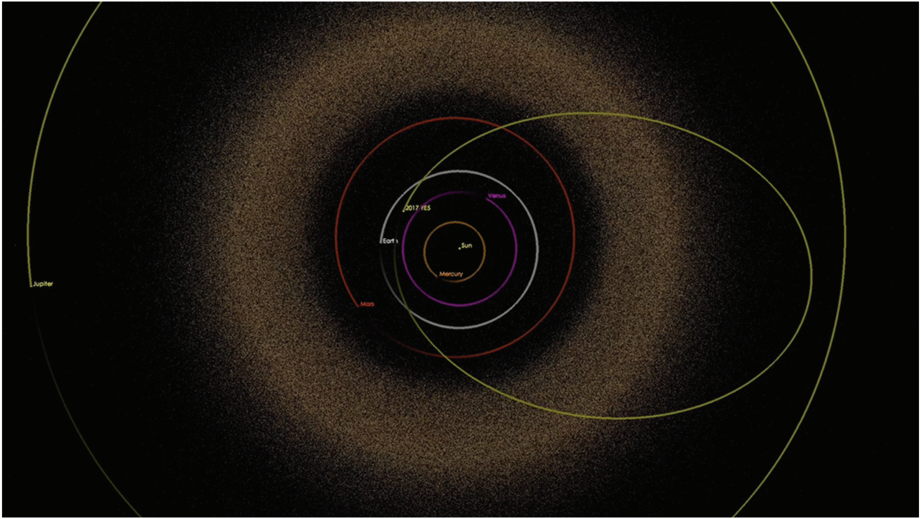
Animation of 2017 YE5 (3)



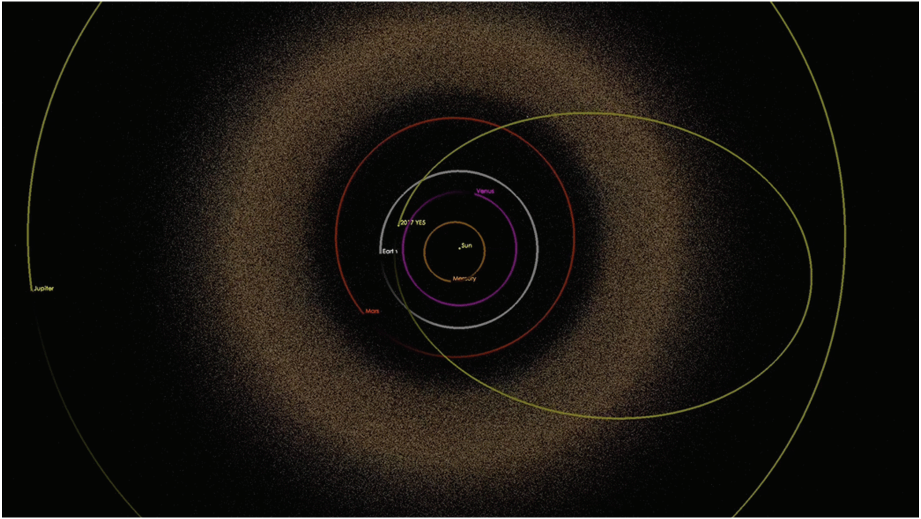
Animation of 2017 YE5 (4)



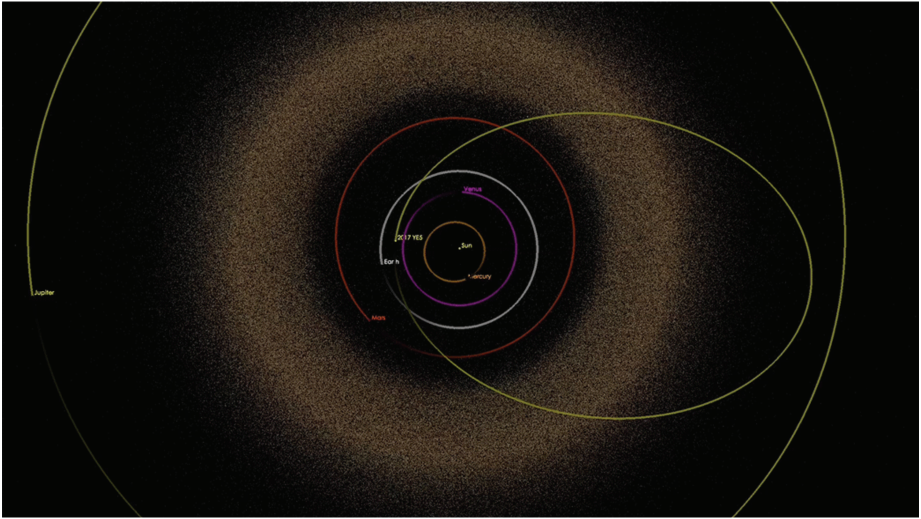
Animation of 2017 YE5 (5)



Animation of 2017 YE5 (6)



Animation of 2017 YE5 (7)



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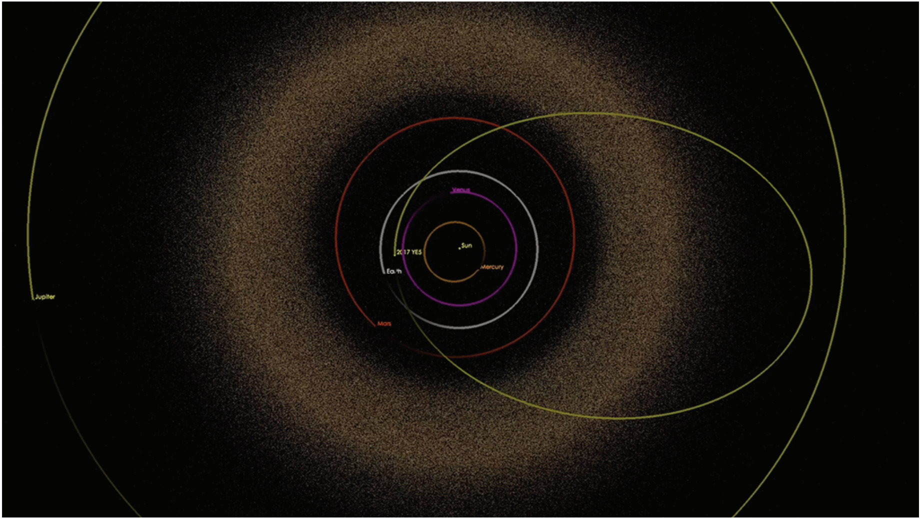
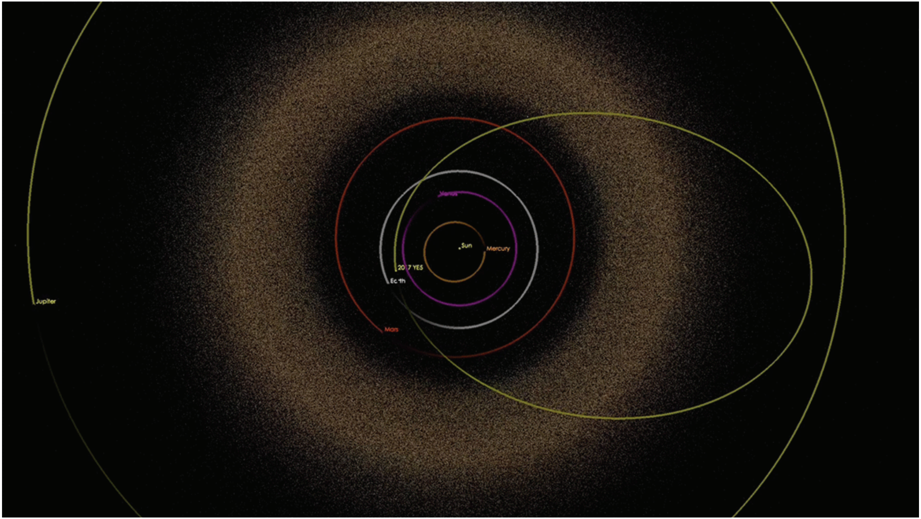


Animation of 2017 YE5 (9)



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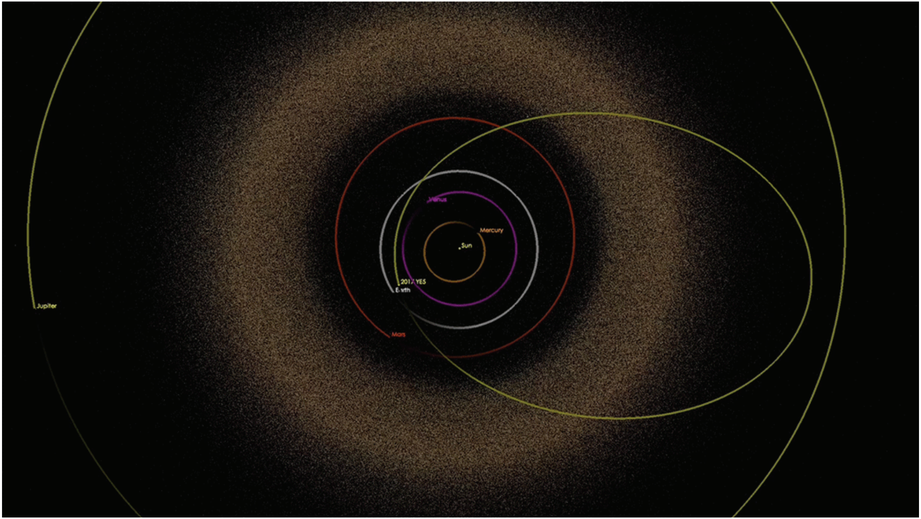

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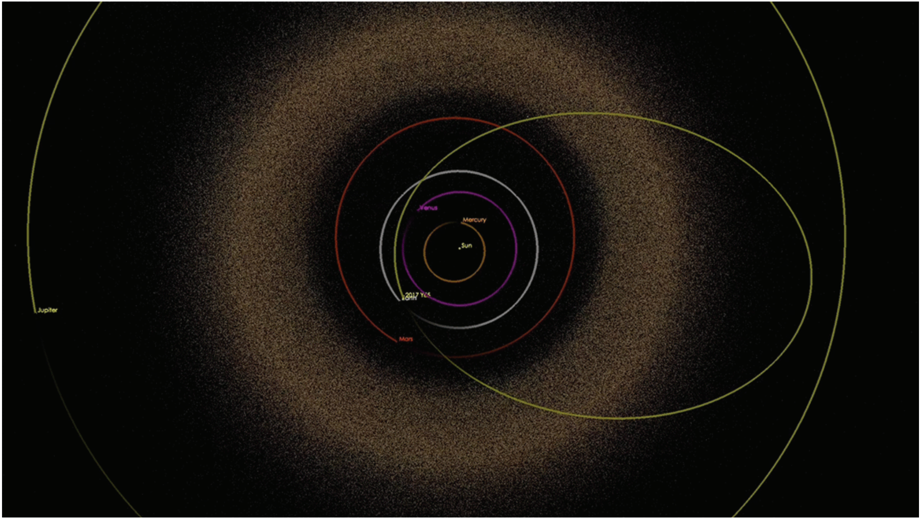

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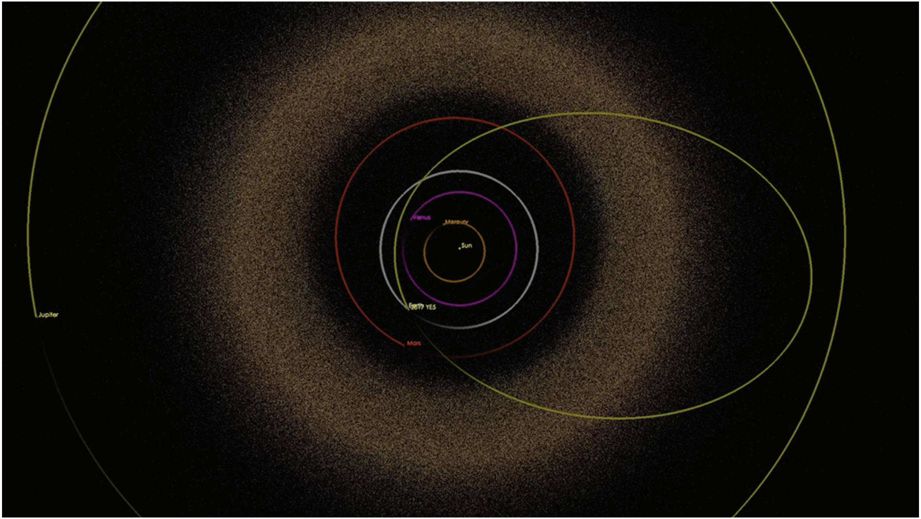
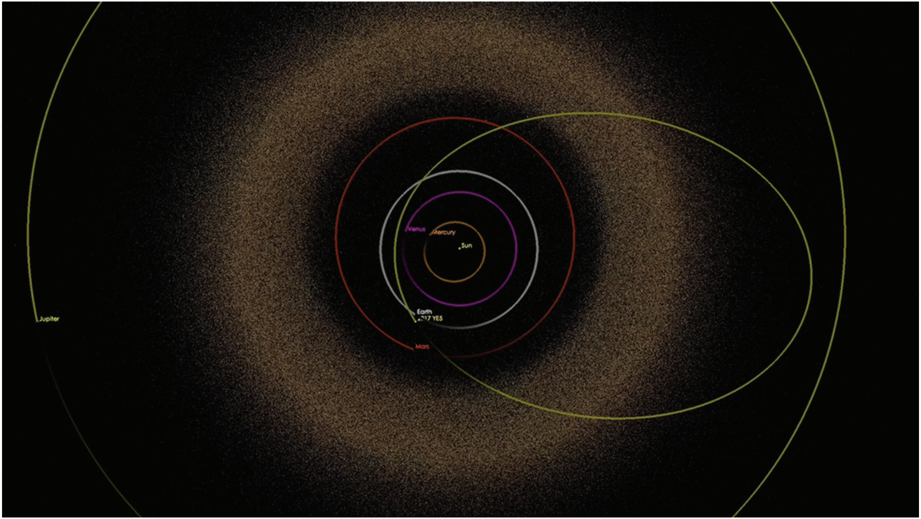
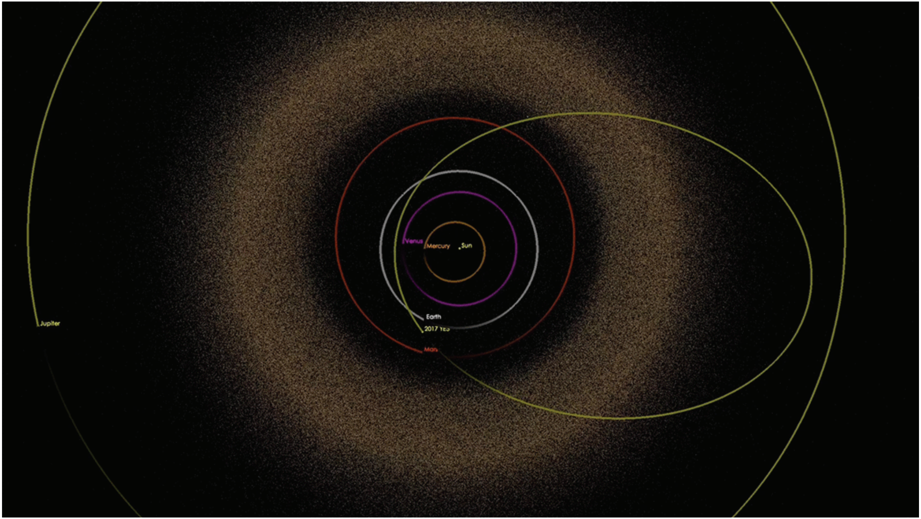


Animation of 2017 YE5 (13)



Animation of 2017 YE5 (14)



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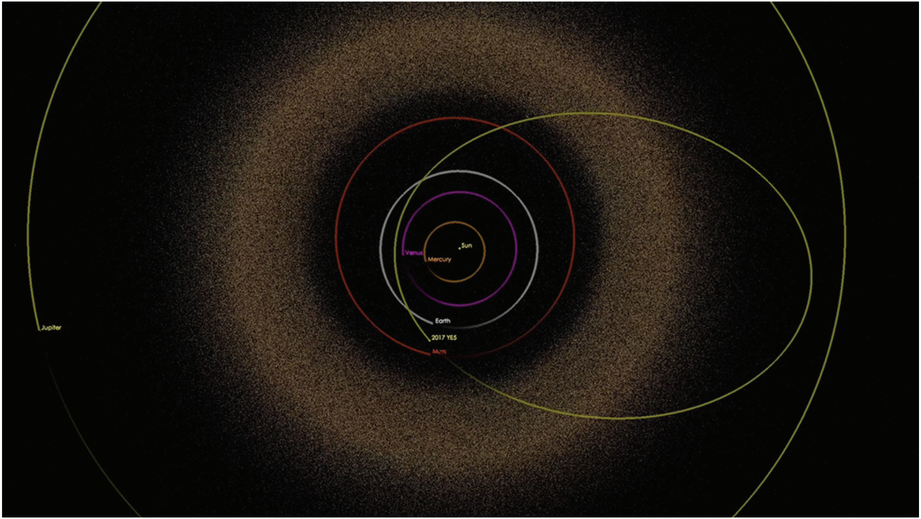
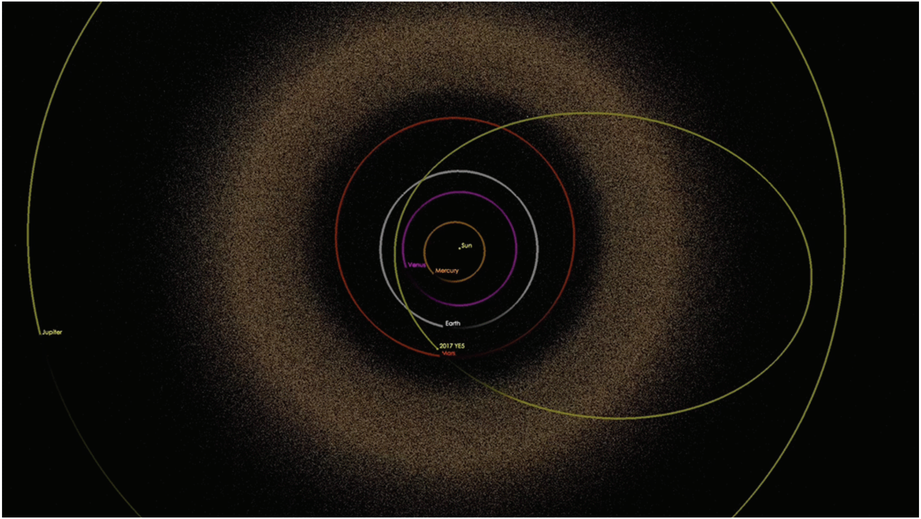


Animation of 2017 YE5 (16)



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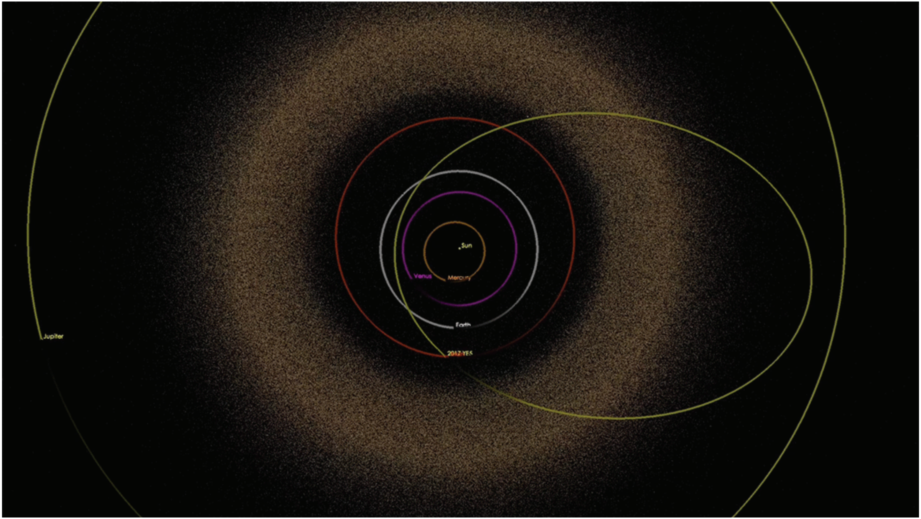
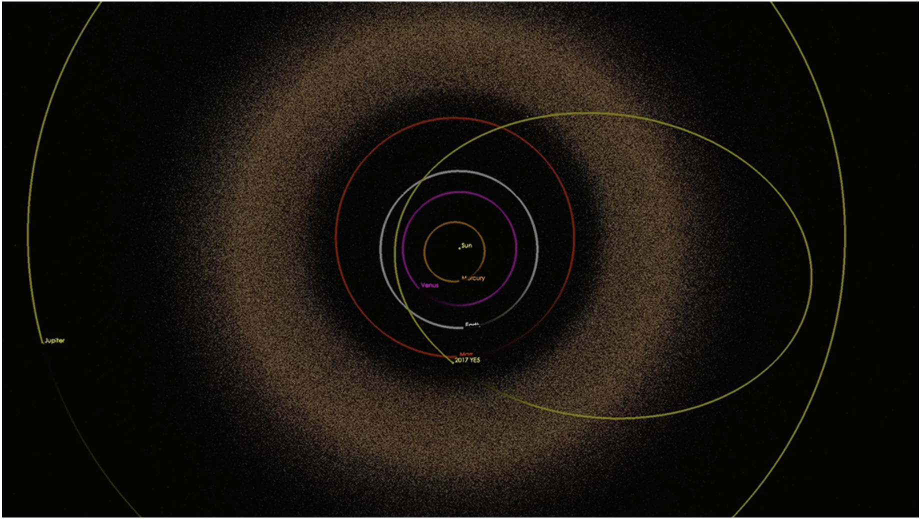
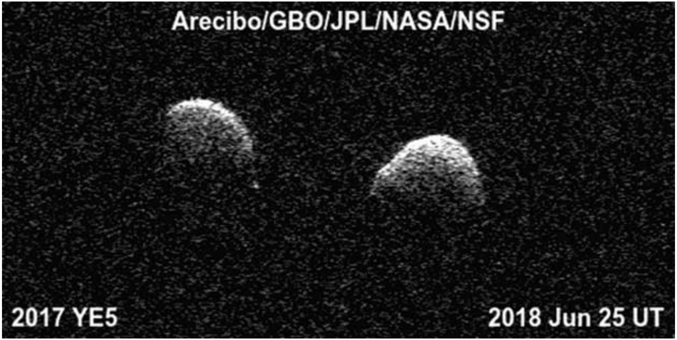


Animation of 2017 YE5 (18)



2017 YE5 is a binary pair



- 2017 YE5 is only the fourth nearly equal mass binary pair, near-Earth asteroid ever detected.
- Each of the two bodies is about 0.9 km in diameter.
- The binary pair revolve about their common barycenter with a period of $20 - 24$ h.

Simplified Four-Body Problem Inspired by 2017 YE5

Hamiltonian for two Primaries at positions $(X1, Y1, 0)$ with mass $M1$, and $(X2, Y2, 0)$ with mass $M2$ is

$$H1 = \frac{P_{X1}^2 + P_{Y1}^2}{2M1} + \frac{P_{X2}^2 + P_{Y2}^2}{2M2} - \frac{M1M2}{\left[(X1 - X2)^2 + (Y1 - Y2)^2\right]^{1/2}}.$$

Hamiltonian $H2 = K2 - U2$ for binary pair at positions $(x1, y1, z1)$ with mass $m1$ and $(x2, y2, z2)$ with mass $m2$ where kinetic energy is

$$K2 = \frac{P_{x1}^2 + P_{y1}^2 + P_{z1}^2}{2m1} + \frac{P_{x2}^2 + P_{y2}^2 + P_{z2}^2}{2m2}$$

and potential energy is

Simplified Four-Body Problem Inspired by 2017 YE5

$$\begin{aligned}U2 = &\frac{m1m2}{[(x2-x1)^2+(y2-y1)^2+(z2-z1)^2]^{1/2}}\\&+\frac{m1M1}{[(X1-x1)^2+(Y1-y1)^2+z1^2]^{1/2}}\\&+\frac{m1M2}{[(X2-x1)^2+(Y2-y1)^2+z1^2]^{1/2}}\\&+\frac{m2M1}{[(X1-x2)^2+(Y1-y2)^2+z2^2]^{1/2}}\\&+\frac{m2M2}{[(X2-x2)^2+(Y2-y2)^2+z2^2]^{1/2}},\end{aligned}$$

where $(X1, Y1, 0)$ and $(X2, Y2, 0)$ are the positions of the two primaries.

This results in a system of 20 first order autonomous differential equations in which the binary pair does not affect the primaries.

Analytic Theory for Structuring IVP

For the planar two-body problem, if r is the difference of the position vectors, then

$$\mu^2(e^2 - 1) = 2hc^2$$

where

- μ is the total mass,
- e is the eccentricity,
- $h = (1/2)(\dot{r} \cdot \dot{r}) - \mu/|r|$ (the total energy), and
- c is the z-component of angular momentum $r \times \dot{r}$.

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- c is the z-component of angular momentum $r \times \dot{r}$.

To keep things simple, we set $e = 0$ and keep the motion of the primaries and the binary pair in the same plane (the inclination of the binary pair is set to 0).

Structured IVP: Circular Motion for Primaries

For primaries with center of mass at the origin, linear momentum 0, total mass $M = M1 + M2$, total energy $h < 0$, and angular momentum $c \neq 0$, the initial conditions are

$$X1 = -\frac{M2 \cdot d \cdot \cos\theta}{M} \qquad P_{X1} = \frac{M1 \cdot M2 \cdot \sin\theta}{\sqrt{Md}}$$

$$Y1 = -\frac{M2 \cdot d \cdot \sin\theta}{M} \qquad P_{Y1} = -\frac{M1 \cdot M2 \cdot \cos\theta}{\sqrt{Md}}$$

$$X2 = \frac{M1 \cdot d \cdot \cos\theta}{M} \qquad P_{X2} = -\frac{M1 \cdot M2 \cdot \sin\theta}{\sqrt{Md}}$$

$$Y2 = \frac{M1 \cdot d \cdot \sin\theta}{M} \qquad P_{Y2} = \frac{M1 \cdot M2 \cdot \cos\theta}{\sqrt{Md}}$$

where d is the distance between the two primaries and $\theta \in [0, 2\pi)$ is angle of the line through the origin that the primaries start on.

This gives four parameters for the circular motion of the two primaries.

Structured IVP: Initial Circular Motion for Binary Pair

Ignoring the primaries/binary pair interactions terms in $U2$, for the binary pair with starting center of mass at $(\xi, 0, 0)$, starting distance b between the binary pair located along x-axis, total mass $m = m1 + m2$, total energy $h < 0$, and angular momentum $c \neq 0$, linear momentum in the x direction 0, and linear momentum in the y direction $u > 0$, the initial conditions for the binary pair are

$$\begin{aligned}
x1 &= (m\xi - m2 \cdot b)/m & P_{x1} &= 0\\
y1 &= 0 & P_{y1} &= m1(u - m2\sqrt{m/b})/m\\
z1 &= 0 & P_{z1} &= 0\\
x2 &= (m\xi + m1 \cdot b)/m & P_{x2} &= 0\\
y2 &= 0 & P_{y2} &= m2(u + m1\sqrt{m/b})/m\\
z2 &= 0 & P_{z2} &= 0.
\end{aligned}$$

This gives five parameters for the initial circular motion of the binary pair.

Numerical Methodology: Search for Stable Motion

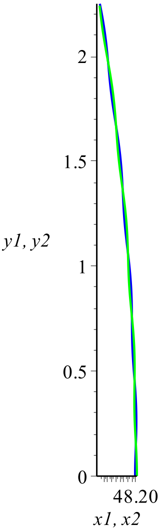
In the search for "stable recurring" motion of the binary pair, fix

- $M1$ (mass of first primary)
- $M2$ (mass of second primary)
- d (constant distance between primaries)
- θ (angle of starting positions)
- $m1$ (mass of one of the binary pair)
- $m2$ (mass of the other of the binary pair)
- ξ (starting position $(\xi, 0, 0)$ of center of mass of binary pair)
- b (starting distance between binary pair)

and vary $u > 0$.

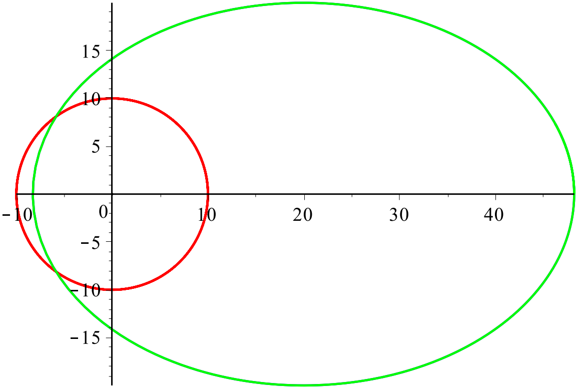
Then plot (1) Initial Binary Pair Motion, (2) Interaction with Primaries, (3) Value of $H2$, and (4) Distance between Binary Pair.

2017 YE5 (Roughly): Initial Motion of Binary Pair



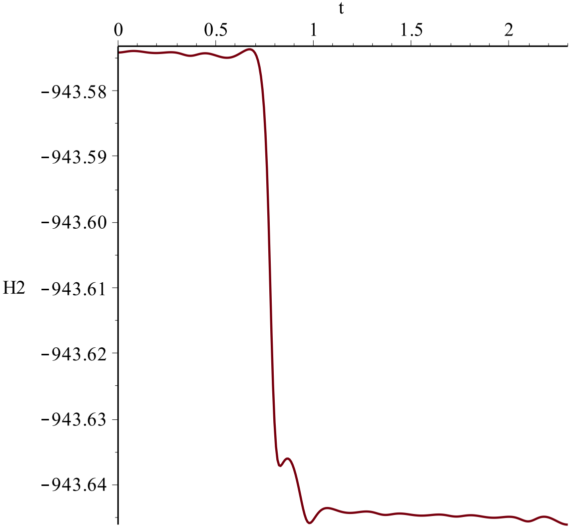
M1=3.329460487 10^5 M2=1 d=10 θ=0 m1=0.08 m2=0.08 ξ=48.2 b= 0.009 u=7.2

2017 YE5 (Roughly): Interaction with Primaries



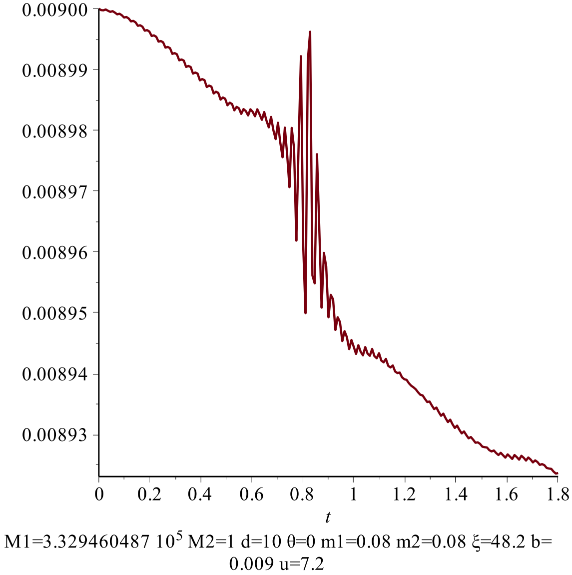
M1=3.329460487 10^5 M2=1 d=10 θ=0 m1=0.08 m2=0.08 ξ=48.2 b=
0.009 u=7.2

2017 YE5 (Roughly): Value of H2

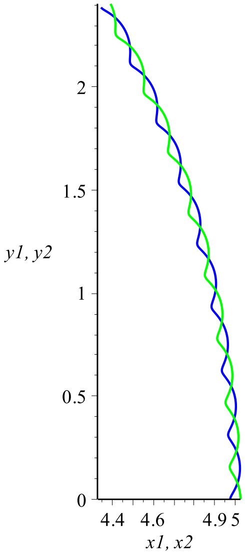


M1=3.329460487 10^5 M2=1 d=10 θ=0 m1=0.08 m2=0.08 ξ=48.2 b= 0.009 u=7.2

2017 YE5 (Roughly): Distance between Binary Pair

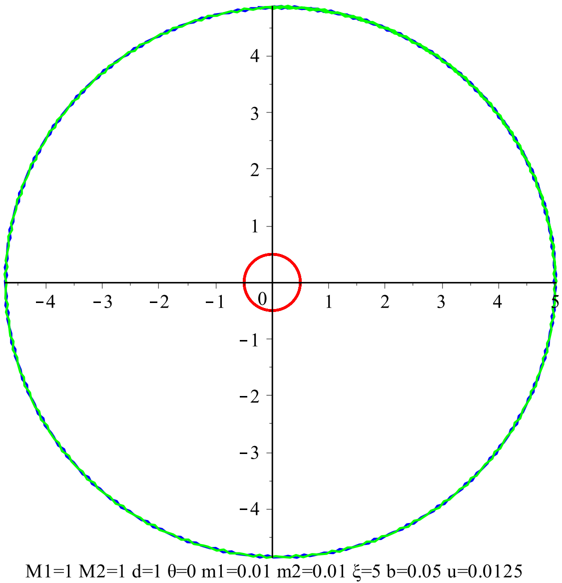


Initial Binary Pair Motion

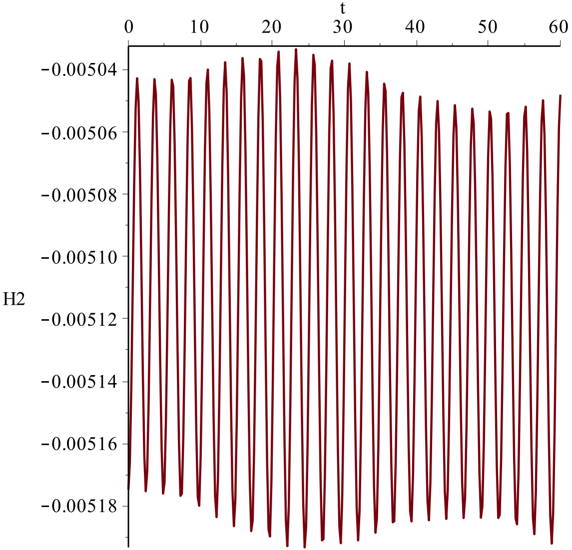


M1=1 M2=1 d=1 θ=0 m1=0.01 m2=0.01 ξ=5 b=0.05 u=0.0125

Nearly Circular Interaction with Equal Mass Primaries

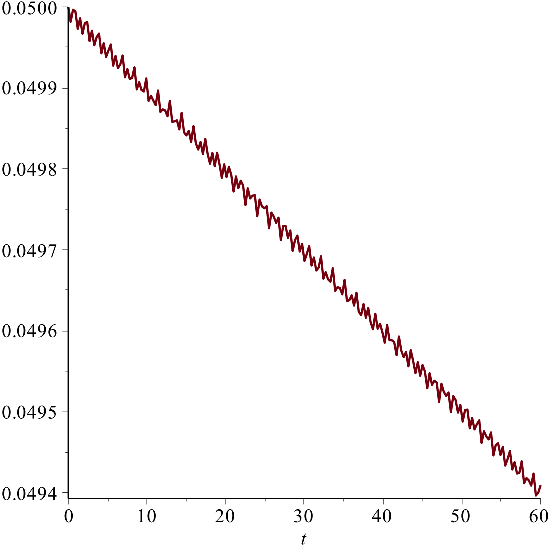


Value of H2



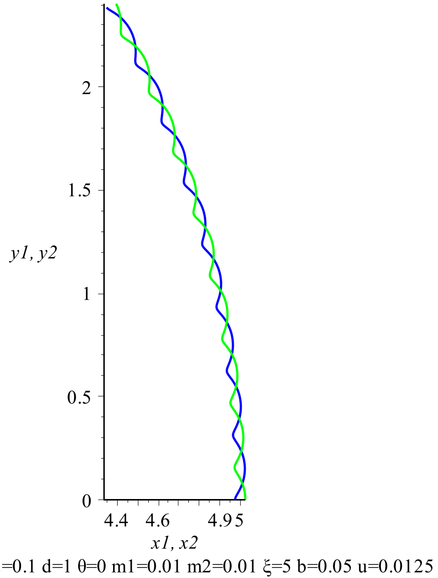
M1=1 M2=1 d=1 θ=0 m1=0.01 m2=0.01 ξ=5 b=0.05 u=0.0125

Distance between Binary Pair

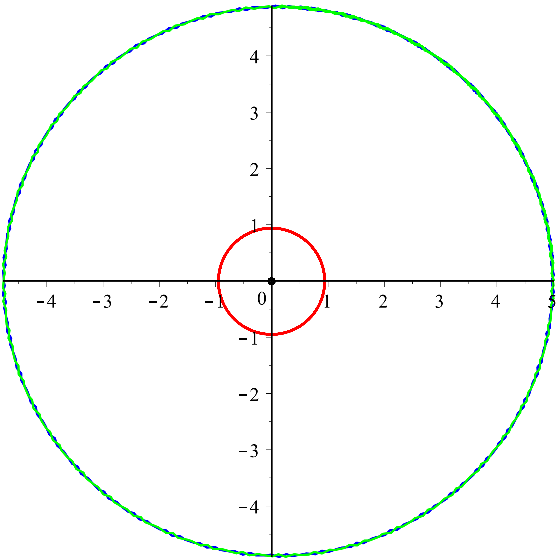


M1=1 M2=1 d=1 θ=0 m1=0.01 m2=0.01 ξ=5 b=0.05 u=0.0125

Initial Binary Pair Motion

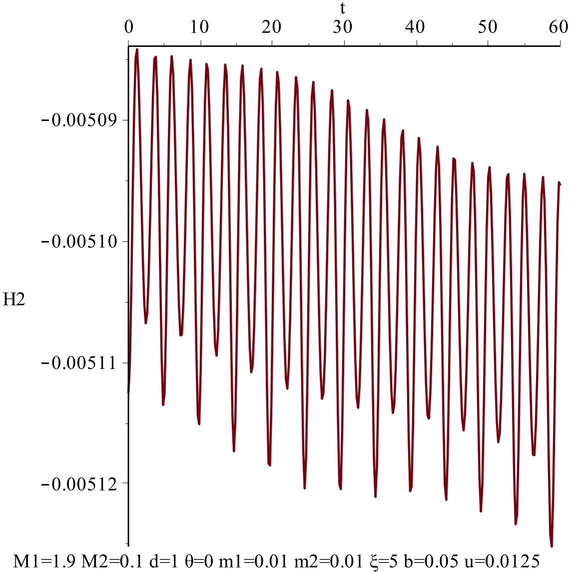


Nearly Circular Interaction with Unequal Mass Primaries



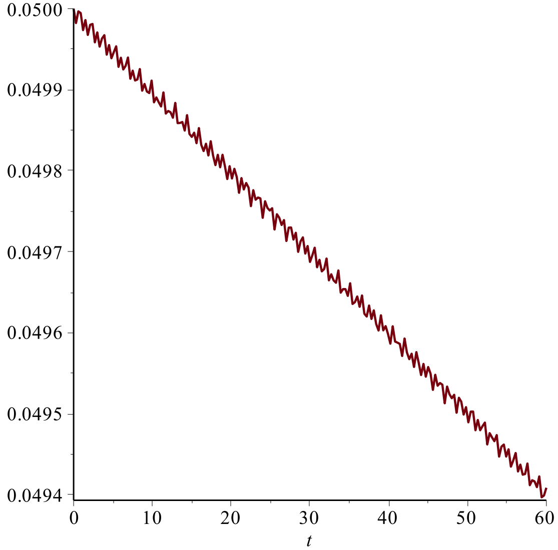
M1=1.9 M2=0.1 d=1 θ=0 m1=0.01 m2=0.01 ξ=5 b=0.05 u=0.0125

Value of H2



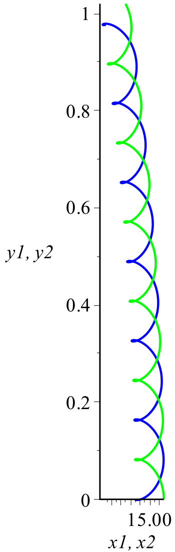
t
0
10
20
30
40
50
60
−0.00509
−0.00510
−0.00511
−0.00512
H2
M1=1.9 M2=0.1 d=1 θ=0 m1=0.01 m2=0.01 ξ=5 b=0.05 u=0.0125

Distance between Binary Pair



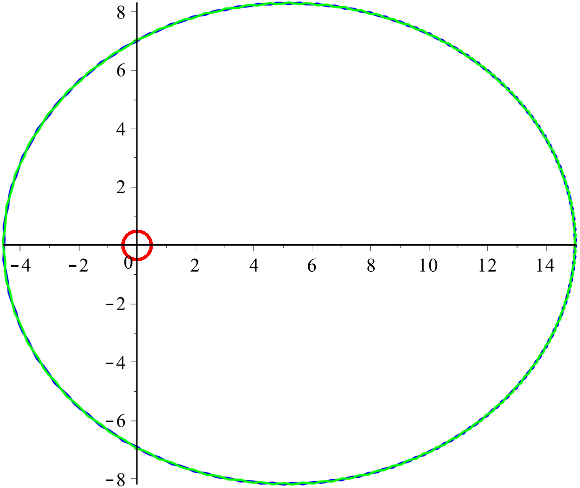
M1=1.9 M2=0.1 d=1 θ=0 m1=0.01 m2=0.01 ξ=5 b=0.05 u=0.0125

Initial Binary Pair Motion



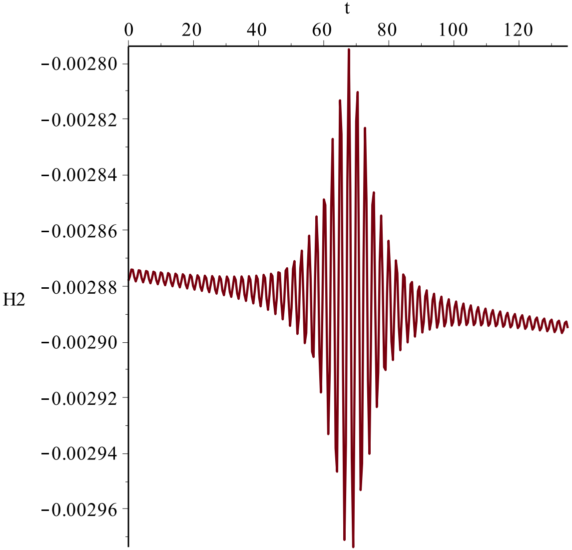
M1=1 M2=1 d=1 θ=0 m1=0.01 m2=0.01 ξ=15 b=0.06 u=0.005

Elliptical Interaction with Equal Mass Primaries



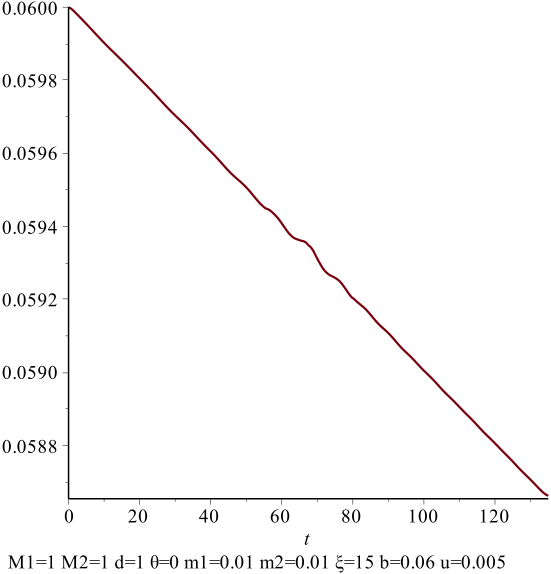
M1=1 M2=1 d=1 θ=0 m1=0.01 m2=0.01 ξ=15 b=0.06 u=0.005

Value of H2

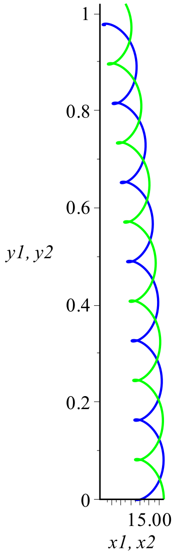


M1=1 M2=1 d=1 θ=0 m1=0.01 m2=0.01 ξ=15 b=0.06 u=0.005

Distance between Binary Pair

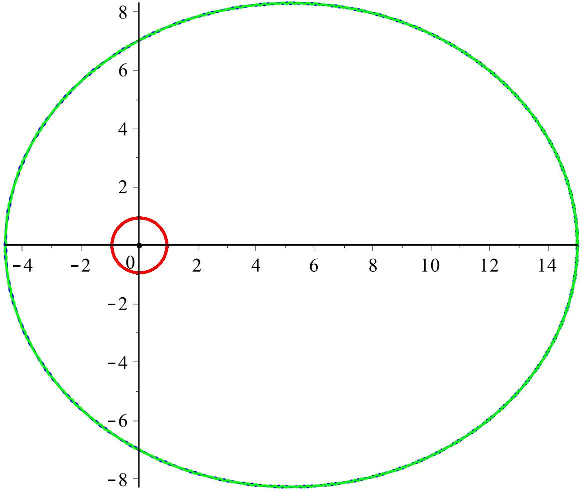


Initial Binary Pair Motion



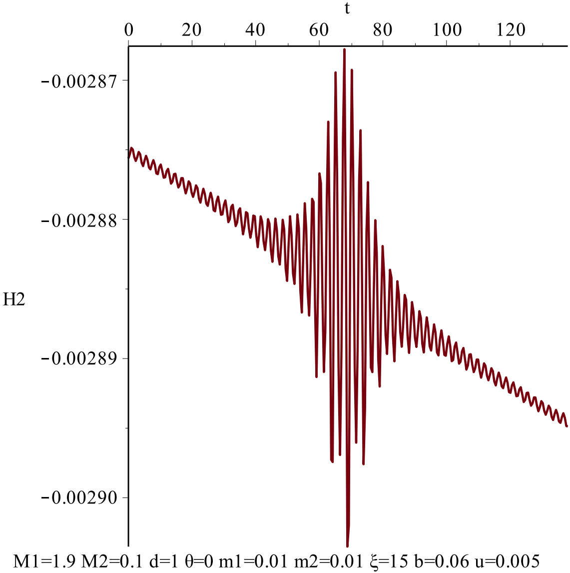
M1=1.9 M2=0.1 d=1 θ=0 m1=0.01 m2=0.01 ξ=15 b=0.06 u=0.005

Elliptical Interaction with Unequal Mass Primaries



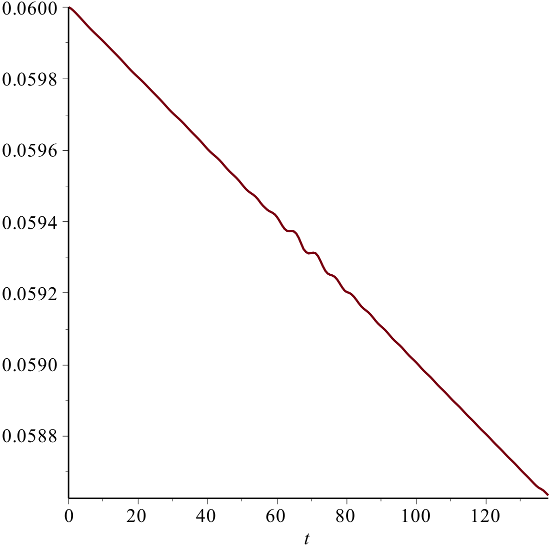
M1=1.9 M2=0.1 d=1 θ=0 m1=0.01 m2=0.01 ξ=15 b=0.06 u=0.005

Value of H2



t
0
20
40
60
80
100
120
H2
−0.00287
−0.00288
−0.00289
−0.00290
M1=1.9 M2=0.1 d=1 θ=0 m1=0.01 m2=0.01 ξ=15 b=0.06 u=0.005

Distance between Binary Pair



M1=1.9 M2=0.1 d=1 θ=0 m1=0.01 m2=0.01 ξ=15 b=0.06 u=0.005

Numerical Methodology: Search for Unstable Motion

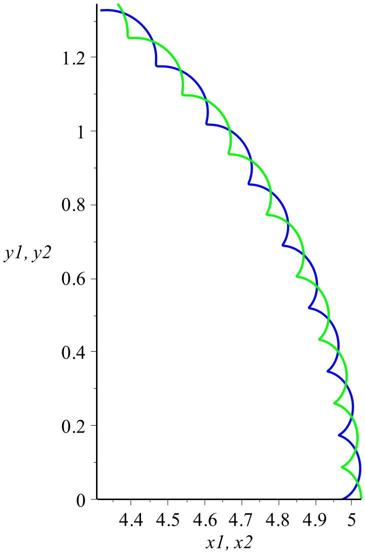
In the search for "unstable" motion of the binary pair, fix

- $M1$ (mass of first primary)
- $M2$ (mass of second primary)
- d (constant distance between primaries)
- $m1$ (mass of one of the binary pair)
- $m2$ (mass of the other of the binary pair)
- ξ (starting position $(\xi, 0, 0)$ of center of mass of binary pair)
- b (starting distance between binary pair)

and vary $u > 0$ and θ to get close interaction of binary pair with primaries.

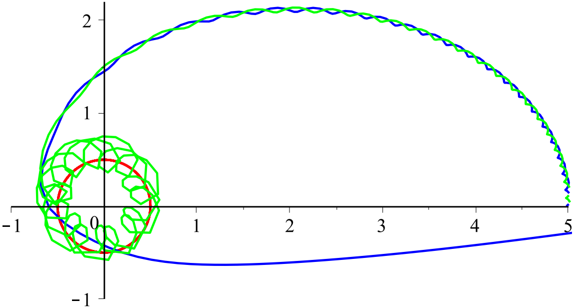
Then plot (1) Initial Binary Pair Motion, (2) Interaction with Primaries, (3) Value of $H2$, and (4) Distance between Binary Pair.

Initial Binary Pair Motion



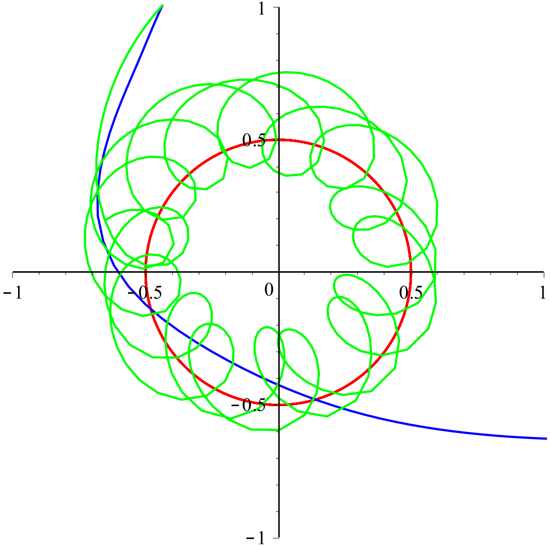
M1=1 M2=1 d=1 θ=0 m1=0.01 m2=0.01 ξ=5 b=0.05 u=0.007

Interaction with Equal Mass Primaries: Capture/Ejection



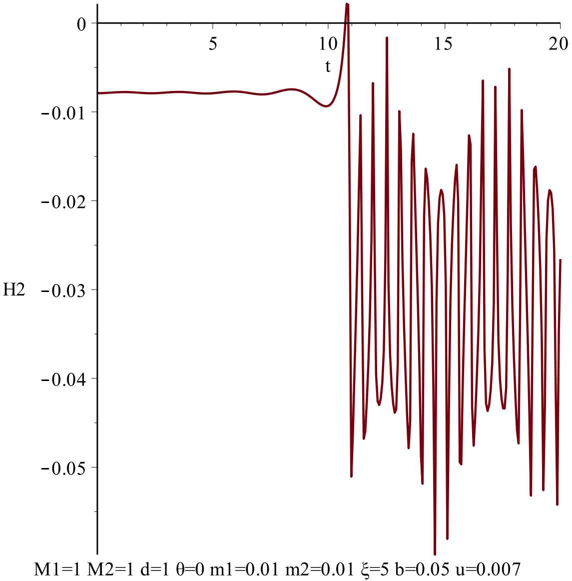
M1=1 M2=1 d=1 θ=0 m1=0.01 m2=0.01 ξ=5 b=0.05 u=0.007

Interaction with Primaries: a closer look

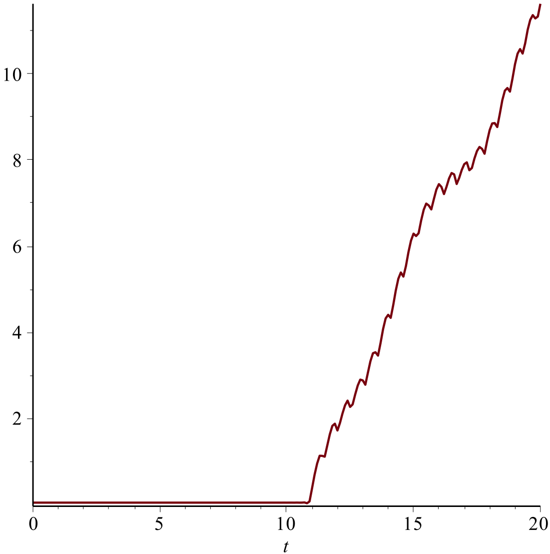


M1=1 M2=1 d=1 θ=0 m1=0.01 m2=0.01 ξ=5 b=0.05 u=0.007

Value of H2

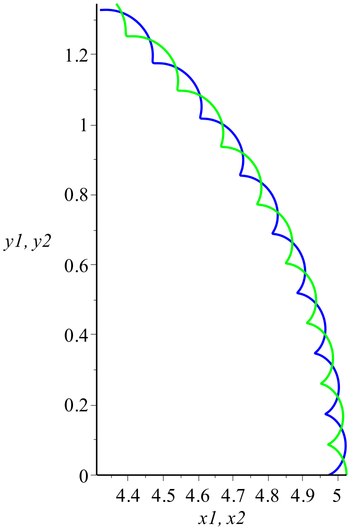

0
5
10
15
20
t
−0.01
−0.02
H2
−0.03
−0.04
−0.05
M1=1 M2=1 d=1 θ=0 m1=0.01 m2=0.01 ξ=5 b=0.05 u=0.007

Distance between Binary Pair



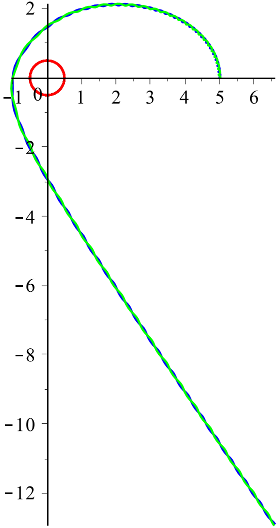
M1=1 M2=1 d=1 θ=0 m1=0.01 m2=0.01 ξ=5 b=0.05 u=0.007

Initial Binary Pair Motion



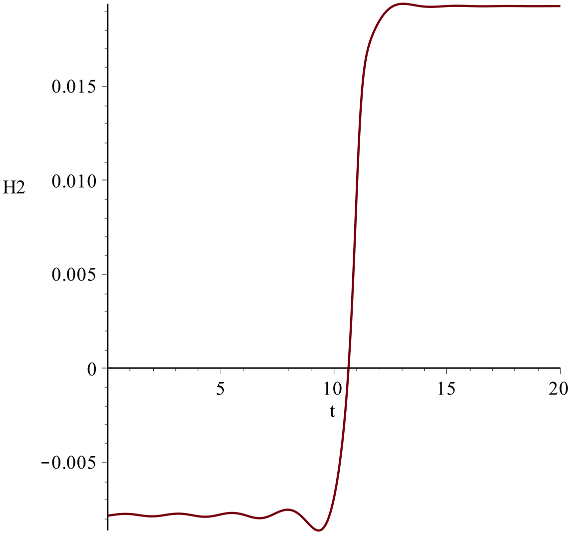
M1=1 M2=1 d=1 $\theta=\frac{1}{6}\pi$ m1=0.01 m2=0.01 ξ=5 b=0.05 u=0.007

Interaction with Equal Mass Primaries: Survival/Ejection



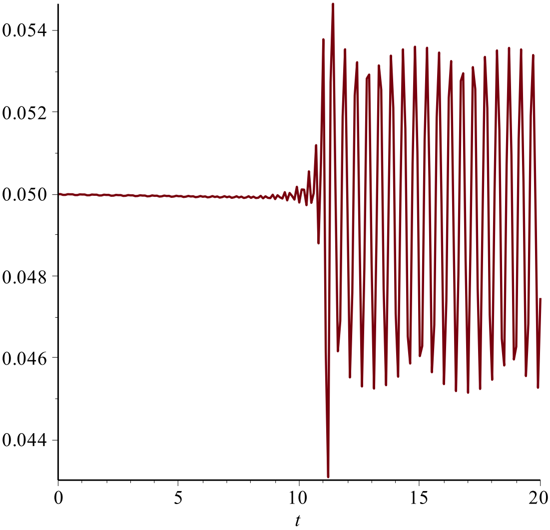
M1=1 M2=1 d=1 $\theta=\frac{1}{6}\pi$ m1=0.01 m2=0.01 ξ=5 b=0.05 u=0.007

Value of H2



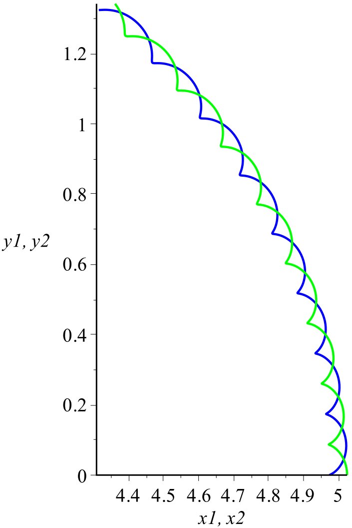
M1=1 M2=1 d=1 $\theta=\frac{1}{6}\pi$ m1=0.01 m2=0.01 ξ=5 b=0.05 u=0.007

Distance between Binary Pair



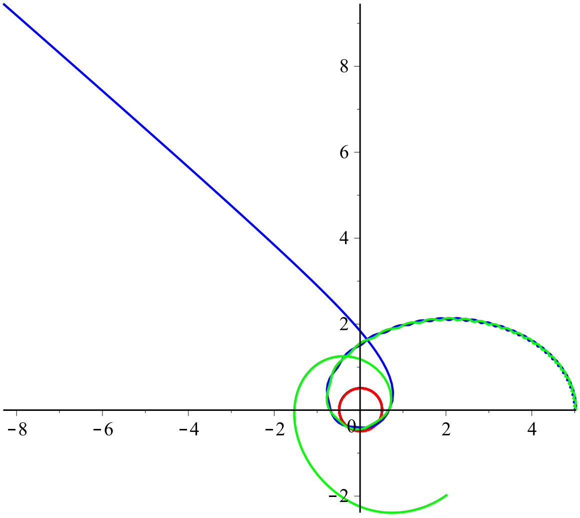
M1=1 M2=1 d=1 $\theta=\frac{1}{6}\pi$ m1=0.01 m2=0.01 ξ=5 b=0.05 u=0.007

Initial Binary Pair Motion

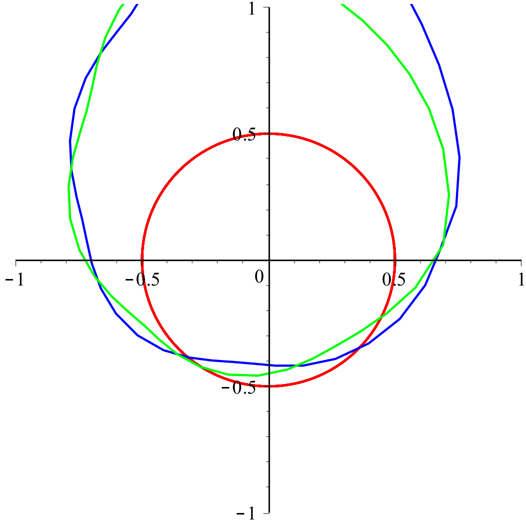


M1=1 M2=1 d=1 $\theta = -\frac{7}{18}\pi$ m1=0.01 m2=0.01 ξ=5 b=0.05 u=0.007

Interaction with Equal Mass Primaries: Separation/Ejection

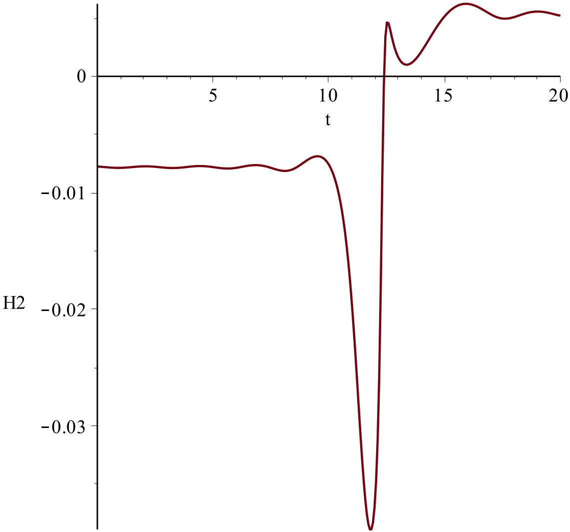


Interaction with Primaries: A closer look



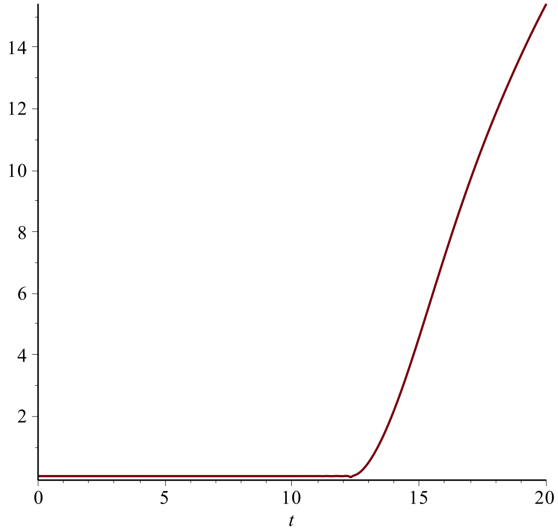
M1=1 M2=1 d=1 $\theta = -\frac{7}{18}\pi$ m1=0.01 m2=0.01 ξ=5 b=0.05 u=0.007

Value of H2



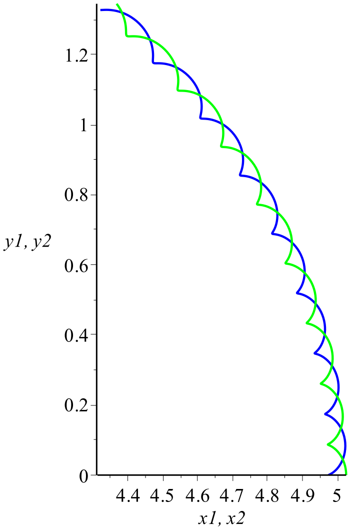
M1=1 M2=1 d=1 $\theta=-\frac{7}{18}\pi$ m1=0.01 m2=0.01 ξ=5 b=0.05 u=0.007

Distance between Binary Pair



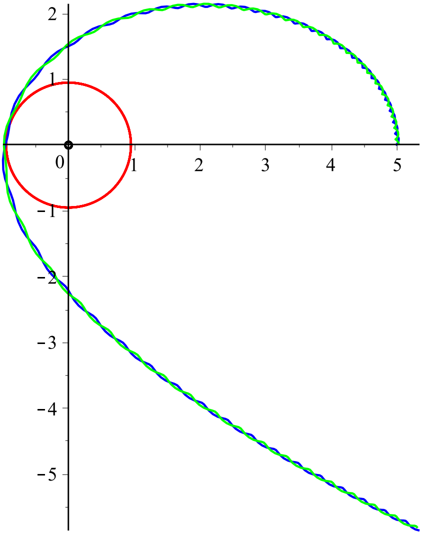
M1=1 M2=1 d=1 $\theta = -\frac{7}{18}\pi$ m1=0.01 m2=0.01 ξ=5 b=0.05 u=0.007

Initial Binary Pair Motion



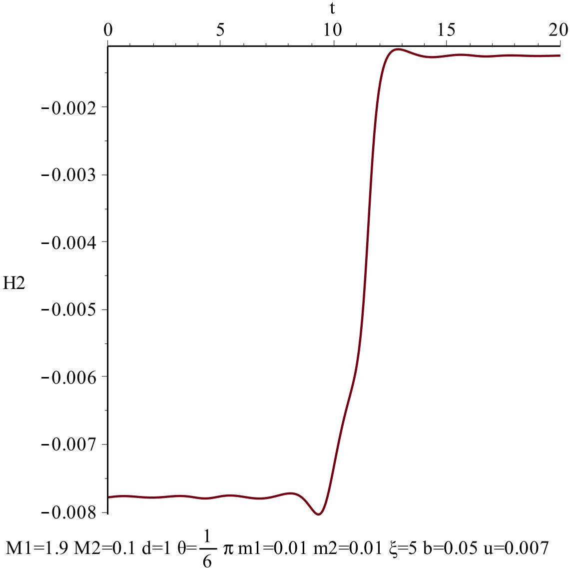
M1=1.9 M2=0.1 d=1 $\theta=\frac{1}{6}\pi$ m1=0.01 m2=0.01 ξ=5 b=0.05 u=0.007

Interaction with Unequal Mass Primaries: Survival



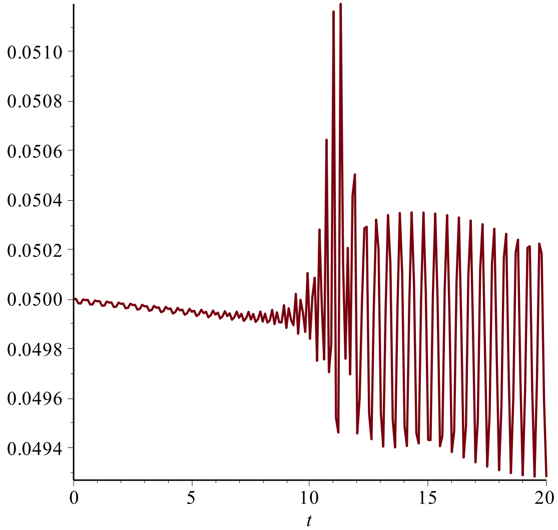
M1=1.9 M2=0.1 d=1 $\theta=\frac{1}{6}\pi$ m1=0.01 m2=0.01 ξ=5 b=0.05 u=0.007

Value of H2



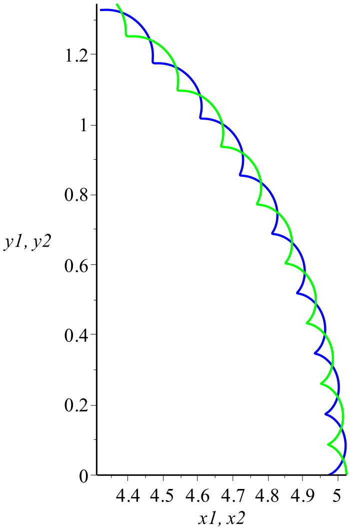
t
0
5
10
15
20
-0.002
-0.003
-0.004
-0.005
-0.006
-0.007
-0.008
H2
M1=1.9 M2=0.1 d=1 θ=1/6 π m1=0.01 m2=0.01 ξ=5 b=0.05 u=0.007

Distance between Binary Pair



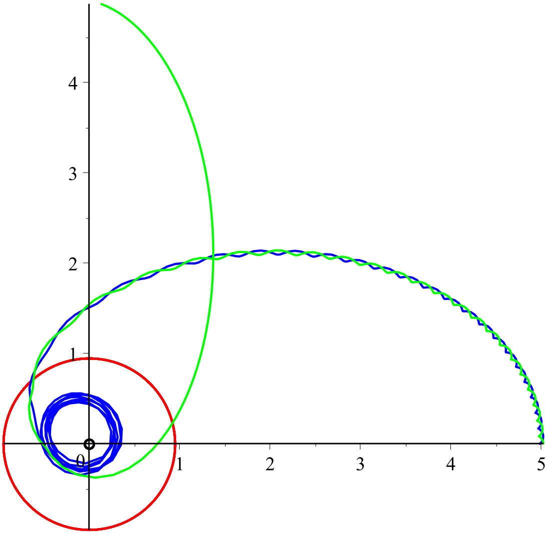
M1=1.9 M2=0.1 d=1 $\theta=\frac{1}{6}\pi$ m1=0.01 m2=0.01 ξ=5 b=0.05 u=0.007

Initial Binary Pair Motion



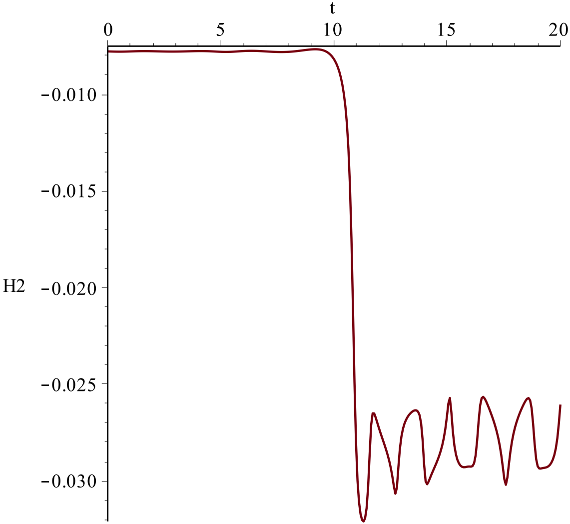
M1=1.9 M2=0.1 d=1 $\theta=-\frac{2}{9}\pi$ m1=0.01 m2=0.01 ξ=5 b=0.05 u=0.007

Interaction with Primaries: Capture/Separation



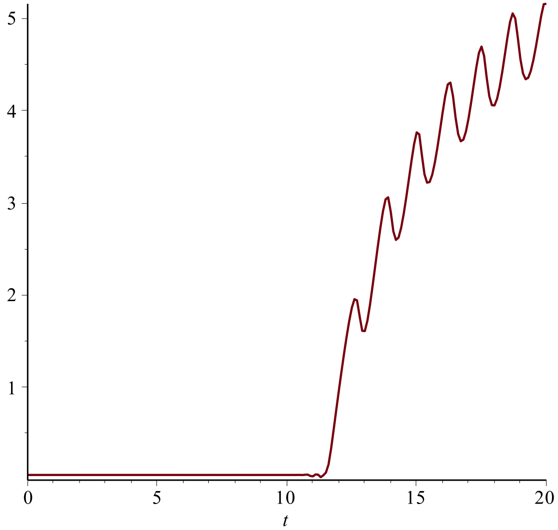
M1=1.9 M2=0.1 d=1 $\theta = -\frac{2}{9}\pi$ m1=0.01 m2=0.01 ξ=5 b=0.05 u=0.007

Value of H2



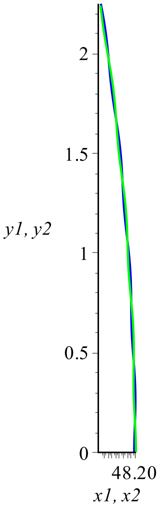
M1=1.9 M2=0.1 d=1 $\theta=-\frac{2}{9}\pi$ m1=0.01 m2=0.01 ξ=5 b=0.05 u=0.007

Distance between Binary Pair



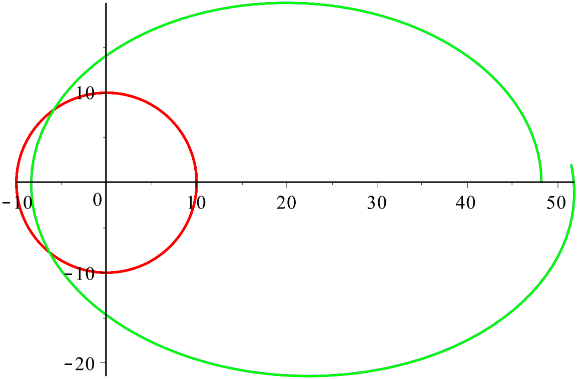
M1=1.9 M2=0.1 d=1 $\theta=-\frac{2}{9}\pi$ m1=0.01 m2=0.01 ξ=5 b=0.05 u=0.007

2017 YE5 (Roughly): Initial Motion of Binary Pair



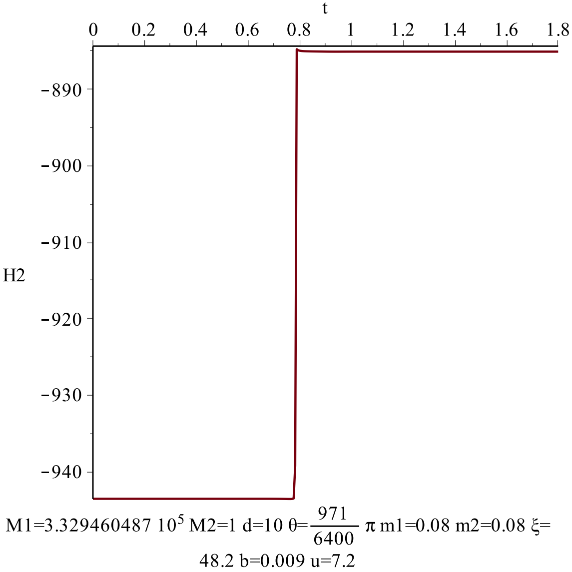
$$M1=3.329460487\ 10^5\ M2=1\ d=10\ \theta=\frac{971}{6400}\pi\ m1=0.08\ m2=0.08\ \xi=48.2\ b=0.009\ u=7.2$$

2017 YE5 (Roughly): Interaction with Primaries, Survival

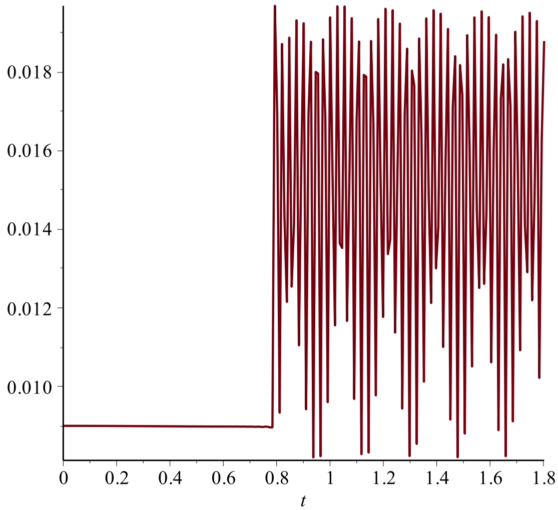


$$M1=3.329460487\ 10^5\ M2=1\ d=10\ \theta=\frac{971}{6400}\pi\ m1=0.08\ m2=0.08\ \xi=48.2\ b=0.009\ u=7.2$$

2017 YE5 (Roughly): Value of H2

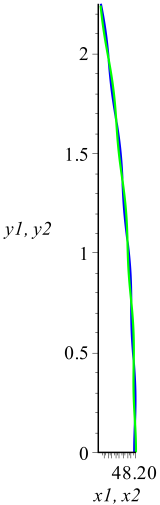


2017 YE5 (Roughly): Distance between Binary Pair



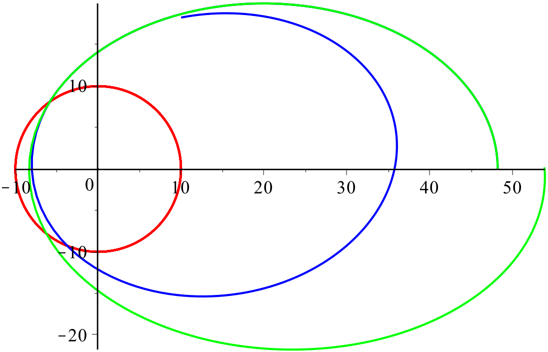
$M1=3.329460487\ 10^5\ M2=1\ d=10\ \theta=\frac{971}{6400}\pi\ m1=0.08\ m2=0.08\ \xi=48.2\ b=0.009\ u=7.2$

2017 YE5 (Roughly): Initial Motion of Binary Pair



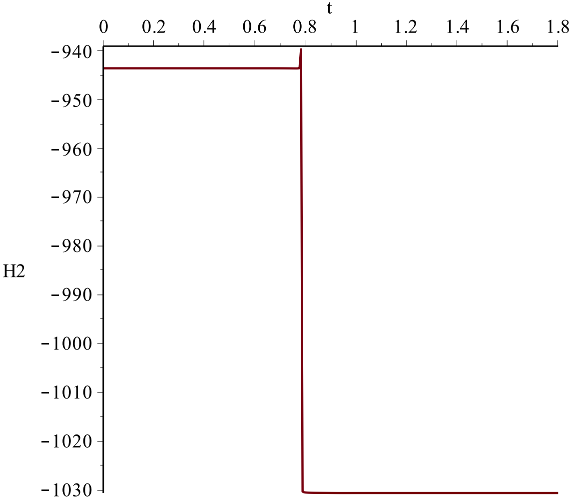
$M1=3.329460487\ 10^5\ M2=1\ d=10\ \theta=\frac{969}{6400}\pi\ m1=0.08\ m2=0.08\ \xi=48.2\ b=0.009\ u=7.2$

2017 YE5 (Roughly): Interaction with Primaries, Separation



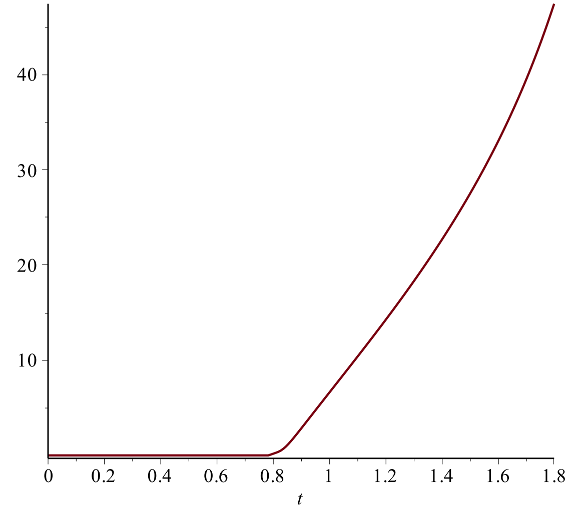
$$M1=3.329460487\ 10^5\ M2=1\ d=10\ \theta=\frac{969}{6400}\pi\ m1=0.08\ m2=0.08\ \xi=48.2\ b=0.009\ u=7.2$$

2017 YE5 (Roughly): Value of H2



M1=3.329460487 10^5 M2=1 d=10 $\theta=\frac{969}{6400}\pi$ m1=0.08 m2=0.08 ξ=48.2 b=0.009 u=7.2

2017 YE5 (Roughly): Distance between Binary Pair



M1=3.329460487 10^5 M2=1 d=10 $\theta=\frac{969}{6400}\pi$ m1=0.08 m2=0.08 ξ= 48.2 b=0.009 u=7.2