Gravitational Singularities

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BYU

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The "Eight" Planet Solar System

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A singular story with several singular characters....

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A singular story with several singular characters....

that weren't always well-behaved....



Tycho Brahe

1546-1601



• At age 20, lost part of his nose in sword duel with third cousin about the legitimacy of a mathematical formula.



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- After wore fake nose made out of metal.



- At age 20, lost part of his nose in sword duel with third cousin about the legitimacy of a mathematical formula.
- After wore fake nose made out of metal.
- Body was exhumed not once, but twice (once in 1901 and then again in 2010) to determine cause of his mysterious death.



• One of the last of naked-eye observational astronomers (before the invitation of telescope in 1608 in Netherlands).

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• One of the last of naked-eye observational astronomers (before the invitation of telescope in 1608 in Netherlands).

• 30 years plus of observations of planetary positions, some of which were by far the most accurate.



Johannes Kepler

1571-1630

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• Began in 1600 to analyze (but forbidden to copy) Tycho's observational data for Mars.

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• Fit the orbital data of Mars to an ellipse (1609, first of Kepler's Three Laws)

Kepler's Second Law (1609)



Sweeping out equal area in equal time

Kepler's Second Law (1609)



Sweeping out equal area in equal time

Kepler's Third Law (1619)

Kepler's Second Law (1609)



Sweeping out equal area in equal time

Kepler's Third Law (1619)

Period Squared proportional to Semi-Major Axis Cubed



Robert Hooke

1635-1703

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• Discovered extension of a spring is proportional to the force (Hooke's Law).

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- Discovered extension of a spring is proportional to the force (Hooke's Law).
- Achieved great wealth through hard work and honesty, only to become jealous later in life.

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• Prior to 1679, came close to to an experimental proof that gravity follows an inverse square law.

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- Prior to 1679, came close to to an experimental proof that gravity follows an inverse square law.
- Communicated to Newton in 1679 inverse square dependence for gravity on distance.



Isaac Newton

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• From 1679 to 1687, formulated universal law of gravitation.



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- In 1686, faced Hooke's accusation of plagiarism, claiming he (Hooke) should be credited as author of inverse square law.



• Made use of inverse square law and his higher order derivatives (Calculus) to formulate the *N*-Body Problem.

$$m_i \ddot{q}_i = \sum_{j \neq i} \frac{Gm_i m_j (q_j - q_i)}{\|q_j - q_i\|^3}, \ i = 1, 2, \dots, N$$



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• Using Calculus, solved 2-body problem.



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- Showed that Kepler's Three *empirical* Laws were a consequence of the *mathematical* model.



Paul Painlevé



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• Prime Minister of France twice (in 1917 and in 1925).



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- As Minister of War from 1925 to 1929, instrumental in the creation and design of Maginot Line.



• Before career in politics, studied how solutions of

$$m_i \ddot{q}_i = \sum_{j \neq i} \frac{Gm_i m_j (q_j - q_i)}{\|q_j - q_i\|^3}, \ i = 1, 2, \dots, N$$

approach the collision (or denominator zero) set

$$\Delta = \bigcup_{j \neq i} \{(q_1, \ldots, q_N) : q_j = q_i\} \subset \mathbb{R}^{3N}.$$

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Depiction of Collision Set





• Already known before Painlevé when solution of 2-body problem approaches collision set, it approaches specific point (a *collision singularity*, the two bodies collide).

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- Already known before Painlevé when solution of 2-body problem approaches collision set, it approaches specific point (a *collision singularity*, the two bodies collide).
- In 1897 for 3-body problem, showed when a solution approaches collision set, it approaches a specific point on collision set (a collision singularity where at least two of the bodies collide).



• Unable to show same for *N*-body problem when $N \ge 4$.



- Unable to show same for *N*-body problem when $N \ge 4$.
- Painlevé conjecture: the *possibility* of a solution approaching the collision without approaching a specific point of collision set (a noncollision singularity).

Two ways to approach the collision set



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1992, Jeff Xia's 5-body problem solution to Painlevé Conjecture



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1992, Jeff Xia's 5-body problem solution to Painlevé Conjecture



2014, Jinxin Xue 4-body problem solution to Painlevé Conjecture

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 \bullet Non-collision singularities have never been observed in the solar system.

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- In the days of the ancient Greeks? Tycho Brahe? Newton? 19th century?

Fragments of Comet Shoemaker-Levy 9, in July 1994



Image: Image:

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- An online news posting March 6, 2012:

"Despite feverish speculation from doomsayers, the near-Earth asteroid 2012 DA14 won't slam into our planet next year, NASA researchers say. The asteroid, which astronomers estimate to be about 150 feet across, will give Earth an uncomfortably close shave on Feb. 15, 2013, coming nearer to our planet than the satellites we've lofted to geostationary orbit."

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• How close of a shave? about 17,000 miles, close to a collision singularity.





Do all collision singularities end in destruction?

Philae touchdown on Comet 67P Churyumov-Gerasimenko Nov. 12, 2014 (First Year Anniversary)

