MATH 473
WINTER 2019
HOMEWORK 13

The quaternion group Q is a group of order 8. It has the following presentation:

$$Q = \langle a, b | a^4 = b^4 = e, a^2 = b^2, b^{-1}ab = a^{-1} \rangle.$$

Every element of Q can be written as one of $e, a, a^2, a^3, b, ab, a^2b, a^3b$, and we have the relations that $ab = ba^3$ and $ba = a^3b$ (in addition to the fact that $a^2 = b^2$).

1. Determine all of the one-dimensional representations of Q (how many are there?), and find the irreducible submodules of $\mathbb{C}Q$ corresponding to them.

2. Determine an irreducible two-dimensional submodule of $\mathbb{C}Q$. (Hint find the eigenspaces of multiplication by a having eigenvalues i and $-i$. Use a technique similar to Homework 11, problem 3. Be careful to note where things differ for the group Q.)

3. Determine the conjugacy classes of the quaternion group $Q = \langle a, b | a^4 = b^4 = e, a^2 = b^2, b^{-1}ab = a^{-1} \rangle$.

4. For each element x of the quaternion group Q determine the order of $C_Q(x)$.