MATH 473
WINTER 2019
HOMEWORK 17

(1) Let χ_1 and χ_2 be distinct irreducible characters, and let $\chi = d_1\chi_1 + d_2\chi_2$ with $d_1, d_2 \in \mathbb{Z}$. Compute $\langle \chi, \chi \rangle$.

(2) Let $U_1, \ldots, U_r$ be a complete set of nonisomorphic irreducible $\mathbb{C}G$-modules, and let χ_i be the character of U_i. Assume

$$V \cong U_1^{c_1} \oplus \cdots \oplus U_r^{c_r}$$

and

$$W \cong U_1^{d_1} \oplus \cdots \oplus U_r^{d_r},$$

where U_i^n is the direct sum of n copies of U_i. Let χ be the character of V and ψ the character of W. Prove that if $\chi = \psi$ then $V \cong W$.

(3) Let G be the subgroup of S_4 generated by the permutations $(1\ 2)$ and $(3\ 4)$. Let V be the permutation $\mathbb{C}G$-module and let W be the regular $\mathbb{C}G$-module. Determine (with proof) whether V and W are isomorphic.

(4) Let χ_0 be the trivial character of G, and let χ_{reg} be the regular character. Prove that

$$\langle \chi_{\text{reg}}, \chi_0 \rangle = 1.$$