MATH 473
WINTER 2019
HOMEWORK 18

1. Let $G = \langle a : a^3 = e\rangle$ be the cyclic group of order 3, $C_1 = \{e\}$, $C_2 = \{a\}$, and $C_3 = \{a^2\}$ its conjugacy classes. Denote by $K_i : G \to \mathbb{C}$ the characteristic function of C_i; i.e., the function that is 1 on C_i and 0 on the rest of G.

   Write the three characteristic functions K_1, K_2, K_3 as linear combinations of the irreducible characters of G.

2. Let $G = S_3$. Let $f : G \to \mathbb{C}$ be defined by $f(e) = 11$, $f(g) = 3$ for any two-cycle g, and $f(h) = 5$ for any three-cycle h. Use inner products to write f as a linear combination of the irreducible characters of G.

3. Let $G = \langle(1, 2), (3, 4)\rangle \subset S_4$. Decompose the permutation module for G into a direct sum of irreducible submodules. (Hint: Find the four irreducible characters of G, and use Proposition 14.26.)

4. Suppose that χ is a character of G and that for all $g \in G$, $\chi(g)$ is an even integer. Prove or disprove: $\chi = 2\psi$ for some character ψ of G.