MATH 473
WINTER 2019
HOMEWORK 22

1. Let V be a $\mathbb{C}G$-module, and let W be a $\mathbb{C}H$-module. Let $\{v_1, \ldots, v_m\}$ be a basis of V, and let $\{w_1, \ldots, w_n\}$ be a basis of W. Define multiplication of $V \otimes W$ by $(g, h) \in G \times H$ by
$$(v_i \otimes w_j)(g, h) = v_i g \otimes w_j h$$
and extend linearly. Prove that for any $v \in V$ and $w \in W$, that
$$(v \otimes w)(g, h) = vg \otimes wh.$$

2. Let $G = S_3$ and let V be the permutation $\mathbb{C}G$-module with basis $\{v_1, v_2, v_3\}$. Let $H = C_2 = \langle a : a^2 = e \rangle$ and let W be the regular $\mathbb{C}H$-module. Compute the matrix of multiplication by $((1, 2, 3), a)$ on $V \otimes W$ with respect to the basis $\{v_i \otimes h : 1 \leq i \leq 3, h \in H\}$.

3. Determine the character table of $S_3 \times S_3$.

4. Suppose that χ is a character of G that is not faithful. Prove that there is some irreducible character ψ of G such that
$$\langle \chi^n, \psi \rangle = 0$$
for every $n \geq 0$.