MATH 473 WINTER 2019 HOMEWORK 25

- 1. Let $G = H \times K$, where H and K are groups, and let χ be an irreducible character of G. Prove that there are irreducible characters ψ of H and ϕ of K so that $\chi \downarrow H = \phi(1)\psi$ and $\chi \downarrow K = \psi(1)\phi$.
- 2. Let G be any group of order pq, where p and q are primes and $p \leq q$. Prove that every irreducible character of G has degree less than or equal to p.
- 3. Let \mathbb{F}_5 be the finite field with five elements. Let

$$G = \left\{ \begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix} : a, b \in \mathbb{F}_5, a \neq 0 \right\}$$

with the group operation of matrix multiplication. Determine the character table of G.

4. Let χ, ψ, θ be irreducible characters of a finite group G. Prove that

 $\langle \chi \psi, \theta \rangle \le \theta(1)$

where 1 is the identity of G.