MATH 473
WINTER 2019
HOMEWORK 27

1. Let $G = S_4$ and let $H = \langle (1\ 2\ 3\ 4), (1\ 3) \rangle \cong D_8$. For each irreducible character ψ of H, write $\psi \uparrow G$ as a sum of irreducible characters of G.

2. Let H be a subgroup of G, let ψ be a character of H, and let χ be a character of G. Prove that
$$(\psi(\chi{\downarrow}H)) \uparrow G = (\psi \uparrow G)\chi.$$
Hint: Write the left side as $\sum d_i \chi_i$, where the χ_i are the irreducible characters of G. Compute d_i by an inner product, and then manipulate the expression using Frobenius reciprocity (twice) to show that the right hand side is given by the same sum.

3. Let H be a subgroup of G. Prove directly from the definition, that for a $\mathbb{C}H$-submodule U of $\mathbb{C}H$,
$$\dim(U \uparrow G) = [G : H] \dim(U).$$

4. Let H be a subgroup of G. Prove that for any character χ of H,
$$(\chi \uparrow G)(e) = [G : H]\chi(e).$$