## MATH 473 WINTER 2019 HOMEWORK 28

1. Suppose that H is a subgroup of G, and let  $\chi_1, \ldots, \chi_k$  be the irreducible characters of G. Let  $\psi$  be an irreducible character of H. Then for some integers  $d_i$ , we have

$$\psi \uparrow G = d_1 \chi_1 + \dots + d_k \chi_k.$$

Prove that

$$\sum_{i=1}^k d_i^2 \le [G:H].$$

- 2. Let  $G = S_4$ , and let  $H = \langle (1 \ 2) \rangle \subset S_4$ . Let  $\psi$  be the trivial character of H. Compute the values of the character  $\psi \uparrow G$ .
- 3. Let G be a finite group, and let  $H = \{e\} \subset G$ . Let  $\psi$  be the trivial character of H (indeed the only character of H). Compute the values of the character  $\psi \uparrow G$ , and write it as a linear combination of the irreducible characters of G.
- 4. Let *H* be a subgroup of *G* with [G : H] = 2, and let  $\psi$  be a character of *H*. Prove that if  $\psi \uparrow G$  is irreducible, then there is exactly one character  $\phi \neq \psi$  of *H* such that  $\phi \uparrow G = \psi \uparrow G$ .