MATH 473
WINTER 2013
HOMEWORK 3

1. Let V be a vector space and let ϑ be an endomorphism of V. Show that ϑ is a projection if and only if there is a basis $\mathscr{B}$ of V such that $[\vartheta]_{\mathscr{B}}$ is a diagonal matrix with all diagonal entries equal to 1 or 0.

2. Let $V = \mathbb{R}^3$, and let $U = \langle (1,2,3),(1,1,1) \rangle$ be a subspace of V. Determine a subspace W of V such that $V = U \oplus W$.

3. let $V = \mathbb{R}^3$, let $U = \langle (1,1,1),(1,1,0) \rangle$ and let $W = \langle (1,0,0) \rangle$. Let $\mathscr{B} = \{(1,0,0),(0,1,0),(0,0,1)\}$ be the standard basis of W. Determine the matrix $[\pi]_{\mathscr{B}}$ of the projection map $\pi : V \to V$ that has U as its image and W as its kernel.

4. Prove or disprove: Let V be a finite dimensional vector space over F, and let U be a subspace. Let $u \in U$, and $v \in V - U$. Prove that there is a projection $\pi : V \to V$ with $\operatorname{Im} \pi = U$ such that $v\pi = u$.