

MATH 473
WINTER 2019
HOMEWORK 30

1. Let p and q be primes with $q > p$, and let G be a nonabelian group of order pq .
 - (a) Determine the degrees of the irreducible characters of G .
 - (b) How many one dimensional characters does G have? (Prove your answer.)
 - (c) Prove that $p|(q-1)$.
 - (d) Determine how many conjugacy classes G has.

2. Let G be a group of odd order, and let 1_G be the trivial character of G .
 - (a) Prove that the only element $g \in G$ with $g = g^{-1}$ is the identity.
 - (b) Suppose that χ is an irreducible character of G with $\chi = \bar{\chi}$. Prove that

$$\langle \chi, 1_G \rangle = \frac{1}{|G|}(\chi(1) + 2a)$$

for some algebraic integer a .

- (c) Prove that every irreducible character χ of G with $\chi = \bar{\chi}$ must satisfy $\chi = 1_G$.
 - (d) Prove that the number of conjugacy classes of G is odd.