MATH 473
WINTER 2019
HOMEWORK 31

1. Let G be a group, and let φ be a character of G such that $\varphi(g) = \varphi(h)$ for all nonidentiry elements g and h of G. Let 1_G be the trivial character of G, and χ_{reg} the regular character of G.
   (a) Show that $\varphi = a1_G + b\chi_{\text{reg}}$ for some $a, b \in \mathbb{C}$.
   (b) Show that $a + b$ and $a + b|G|$ are integers.
   (c) Show that if χ is a nontrivial irreducible character of G, then $b\chi(1)$ is an integer.
   (d) Show that a and b are integers.

2. Prove that every group of order 15 is abelian.

3. Let G be a nonabelian group of order 81. What possibilities are there for the degrees of the irreducible characters of G.