MATH 473 WINTER 2019 HOMEWORK 32

- 1. Without calculating the character table, prove that all irreducible characters of the dihedral group $D_{12} = \langle a, b | a^6 = b^2 = e, b^{-1}ab = a^{-1} \rangle$ have only integer values.
- 2. Use Theorem 22.16 and its converse (stated on page 254) to determine all values of $n \ge 2$ for which all irreducible characters of the dihedral group

$$D_{2n} = \langle a, b | a^n = b^2 = e, b^{-1}ab = a^{-1} \rangle$$

have only integer values.

- 3. Prove that if G is a group of odd order, then no non-identity element of G is real.
- 4. Let G be a finite abelian group. Use the description of the irreducible characters of G given in Theorem 9.8 to prove directly that the number of real irreducible characters of G is equal to the number of elements $g \in G$ for which $g^2 = 1$.