

MATH 473
WINTER 2019
HOMEWORK 33

Feel free to use a computer on any of these problems. Throughout this assignment, $G = \langle a, b : a^4 = e, a^2 = b^2, b^{-1}ab = a^{-1} \rangle$ is the quaternion group of order 8, and V will be a two-dimensional irreducible $\mathbb{C}G$ -module (which you will find in problem 1).

1. Find a two-dimensional irreducible $\mathbb{C}G$ -module V (describe it by giving a basis v_1, v_2 , and the products $v_i a$ and $v_i b$).
2. Let V be the $\mathbb{C}G$ -module described in problem 1. Determine a basis of $V_{\mathbb{R}}$, and find the matrices of multiplication by a , b , and ab with respect to this basis.
3. Let $u \in V_{\mathbb{R}}$ be any nonzero vector in $V_{\mathbb{R}}$. Prove that the elements u, ua, ub, uab are linearly independent and that $V_{\mathbb{R}}$ is an irreducible $\mathbb{R}G$ -module.
4. Compute the character χ of V , and prove that there is no two-dimensional irreducible $\mathbb{R}G$ -module with character χ .