**MATH 473**
**WINTER 2019**
**HOMEWORK 35**

1. The character table of $S_5$ is:

| | $e$ | (1 2) | (1 2)(3 4) | (1 2 3) | (1 2 3 4) | (1 2 3 4 5) | (1 2)(3 4 5) |
|---|---|---|---|---|---|---|---|
| $\lvert x^G \rvert$ | 1 | 10 | 15 | 20 | 30 | 24 | 20 |
| $\chi_1$ | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| $\chi_2$ | 1 | $-1$ | 1 | 1 | $-1$ | 1 | $-1$ |
| $\chi_3$ | 4 | 2 | 0 | 1 | 0 | $-1$ | $-1$ |
| $\chi_4$ | 4 | $-2$ | 0 | 1 | 0 | $-1$ | 1 |
| $\chi_5$ | 5 | 1 | 1 | $-1$ | $-1$ | 0 | 1 |
| $\chi_6$ | 5 | $-1$ | 1 | $-1$ | 1 | 0 | $-1$ |
| $\chi_7$ | 6 | 0 | $-2$ | 0 | 0 | 1 | 0 |

For each irreducible character $\chi_i$ of $S_5$, determine $\iota\chi_i$.

2. Let $G = S_5$. Use corollary 23.17 to determine
$$|\{y \in G : y^2 = (1\ 2\ 3)\}|.$$
Confirm your answer by listing the elements whose square is (1 2 3).

3. Prove that every irreducible character of every group $D_{2n}$ can be realized over the real numbers. (The character tables of these groups are given in chapter 18).

4. Let $\rho$ be an irreducible representation of degree 2 of a group $G$ and let $\chi$ be the character of $\rho$. Prove that $\chi_A(g) = \det(g\rho)$ for all $g \in G$. Prove also that $\iota\chi = -1$ if and only if $\det(g\rho) = 1$ for all $g \in G$.