MATH 473
WINTER 2019
HOMEWORK 4

1. Let G be the cyclic group of order m, say $G = \langle a : a^m = 1 \rangle >$ and let $A \in \mathrm{GL}(n, \mathbb{C})$. Define $\rho : G \to \mathrm{GL}(n, \mathbb{C})$ by
$$(a^r)\rho = A^r,$$
for $0 \leq r < m$. Prove that ρ is a representation of G if and only if $A^m = 1$.

2. Prove that equivalence of representations is an equivalence relation.

3. Prove that if $\rho : G \to \mathrm{GL}(1, \mathbb{C})$ is a representation, then $G/\operatorname{Ker} \rho$ is an abelian group.

4. Find two nonequivalent faithful degree two representations of the cyclic group C_2 with two elements. Be sure to prove that the representations you find are nonequivalent.