MATH 473
WINTER 2019
HOMEWORK 5

1. Let $G = S_n$ and let V be an m-dimensional vector space over F (where $F = \mathbb{R}$ or $\mathbb{C}$. Then (see problem 4.2) V becomes an FG-module if we define
$$vg = \begin{cases} v & \text{if } g \text{ is an even permutation,} \\ -v & \text{if } g \text{ is an odd permutation.} \end{cases}$$
Let B be a basis for V. Describe the representation ρ corresponding to the FG-module V and the basis B.

2. Consider $G = C_4$ as the subgroup of S_4 generated by the permutation $(1\ 2\ 3\ 4)$. Describe the representation corresponding to the permutation module of G over $F = \mathbb{C}$ with respect to the natural basis.

3. With the same notation as problem 2, let $\{v_1, v_2, v_3, v_4\}$ be the natural basis. Describe the representation of G corresponding to the permuation module of G over $F = \mathbb{C}$ with respect to the basis
$\{v_1 + v_2 + v_3 + v_4, v_1 + iv_2 - v_3 - iv_4, v_1 - v_2 + v_3 - v_4, v_1 - iv_2 - v_3 + iv_4\}$.

4. Let $G = C_3 = \langle a : a^3 = 1\rangle$, and let $\rho : G \to \mathrm{GL}(2, F)$ be the representation defined by
$$a\rho = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix},$$
(see Example 5.5(1)). Prove that ρ is irreducible if $F = \mathbb{R}$, but that ρ is reducible if $F = \mathbb{C}$.
Hint: What are the dimensions of any possible subspaces of the FG-module arising from ρ, and how do such submodules relate to eigenvalues of $a\rho$?