MATH 473 WINTER 2019 HOMEWORK 6

- 1. Let $G = S_3$, and write $\alpha = (1 \ 2)$ and $\beta = (1 \ 2 \ 3)$. Write $r = \frac{1}{2}\beta + \frac{1}{2}\beta^2$, and $s = \frac{1}{3}\alpha + \frac{1}{3}\alpha\beta + \frac{1}{3}\alpha\beta^2$. Compute the following products in the group algebra $\mathbb{C}G$. (a) rs. (b) r^2 . (c) s^2 .
- 2. Let G be a finite group that is not the trivial group. Prove that $\mathbb{C}G$ contains zero divisors.

Hint: Let $c = \sum_{g \in G} g$.

- 3. Let ρ and σ be equivalent representations of the group G over F. Prove that ρ is reducible if and only if σ is reducible.
- 4. Let $G = S_3$ and let $\rho : G \to GL(6, \mathbb{C})$ be the regular representation of G. Determine the matrix $(1 \ 2 \ 3)\rho$. (Note: your answer will depend on the ordering that you use for the standard basis of FG, so be sure to specify this ordering.)