MATH 473
WINTER 2019
HOMEWORK 7

1. Let V be an FG-module. Prove that
$$V_0 = \{v \in V : vg = v \text{ for all } g \in G\}$$
is an FG-submodule of V.

2. Assume that F is $\mathbb{R}$ or $\mathbb{C}$. Define $\vartheta : V \to V$ by
$$v\vartheta = \frac{1}{|G|}\sum_{g\in G} vg.$$
Prove that ϑ is an FG-homomorphism and a projection of V with image equal to V_0.

3. Let G be the subgroup of S_4 generated by the permutations $(1\ 2)$ and $(3\ 4)$. Let V be the permutation module for G over $F = \mathbb{C}$. Determine V_0 (defined as in problem 1).

4. Let G be the subgroup of S_4 generated by the permutations $(1\ 2)$ and $(3\ 4)$, and let V be the regular FG-module (with $F = \mathbb{C}$). Determine V_0.