MATH 473
WINTER 2019
HOMEWORK 9

1. Let G be a cyclic group of order 4. Write the regular $\mathbb{C}G$-module as a direct sum of irreducible $\mathbb{C}G$-modules.

2. Let $G = C_2 = \langle a : a^2 = e \rangle$, and let $\rho : G \to \mathrm{GL}(3, \mathbb{C})$ be the representation defined by
$$a\rho = \begin{bmatrix} 3 & 0 & -2 \\ 0 & 1 & 0 \\ 4 & 0 & -3 \end{bmatrix}.$$
Let V be $\mathbb{C}^3$, and make V into a $\mathbb{C}G$-module by defining $vg = v \cdot (g\rho)$ for $g \in G$.
Set $U = \mathrm{span}([0, 1, 0], [-1, 0, 1])$. U is then a $\mathbb{C}G$-submodule of V. Determine two distinct $\mathbb{C}G$-submodules W_1 and W_2 such that
$$V = U \oplus W_1 = U \oplus W_2.$$
This shows that the submodule W determined by Maschke's theorem need not be uniquely defined.

3. Let G be the infinite group
$$G = \left\{ \begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix} : n \in \mathbb{Z} \right\}.$$
Let $V = \mathbb{C}^2$ be a $\mathbb{C}G$-module with the multiplication vg given by matrix multiplication of the row vector $v \in V$ by the matrix $g \in G$. Show that V is **not** completely reducible.

4. Prove that if G is a finite simple group, then every nontrivial irreducible $\mathbb{C}G$-module is faithful.