MATH 473
FALL 2019
HOMEWORK 6

1. Let $G = C_3 = \langle a : a^3 = 1\rangle$, and let $\rho : G \to \mathrm{GL}(2, F)$ be the representation defined by
$$a\rho = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix},$$
(see Example 5.5(1)). Prove that ρ is irreducible if $F = \mathbb{R}$, but that ρ is reducible if $F = \mathbb{C}$.
Hint: What are the dimensions of any possible subspaces of the FG-module arising from ρ, and how do such submodules relate to eigenvalues of $a\rho$?

2. Let ρ and σ be equivalent representations of the group G over F. Prove that ρ is reducible if and only if σ is reducible.

3. Let $G = S_3$, and write $\alpha = (1\ 2)$ and $\beta = (1\ 2\ 3)$. Write $r = \frac{1}{2}\beta + \frac{1}{2}\beta^2$, and $s = \frac{1}{3}\alpha + \frac{1}{3}\alpha\beta + \frac{1}{3}\alpha\beta^2$. Compute the following products in the group algebra $\mathbb{C}G$.
(a) rs.
(b) r^2.
(c) s^2.

4. Let $G = S_3$ and let $\rho : G \to GL(6, \mathbb{C})$ be the regular representation of G. Determine the matrix $(1\ 2\ 3)\rho$. (Note: your answer will depend on the ordering that you use for the standard basis of FG, so be sure to specify this ordering.)