MATH 473 FALL 2019 HOMEWORK 7

1. Let V be an FG-module. Prove that

$$V_0 = \{ v \in V : vg = v \text{ for all } g \in G \}$$

is an FG-submodule of V.

2. Assume that F is \mathbb{R} or \mathbb{C} . Define $\vartheta: V \to V$ by

$$v\vartheta = \frac{1}{|G|} \sum_{g \in G} vg.$$

Prove that ϑ is an *FG*-homomorphism and a projection of *V* with image equal to V_0 .

- 3. Let G be the subgroup of S_4 generated by the permutations (1 2) and (3 4). Let V be the permutation module for G over $F = \mathbb{C}$. Determine V_0 (defined as in problem 1).
- 4. Let G be the subgroup of S_4 generated by the permutations (1 2) and (3 4), and let V be the regular FG-module (with $F = \mathbb{C}$). Determine V_0 .