MATH 473
FALL 2019
HOMEWORK 12

1. Use representation theory to prove that any group of order 4 is abelian.

2. Let $D_{10} = \langle a, b : a^5 = b^2 = e, b^{-1}ab = a^{-1} \rangle$. Prove that D_{10} has exactly two one-dimensional representations.
   Hint: The element a must map to a fifth root of 1.

3. Determine the number and dimensions of a complete set of nonisomorphic irreducible $\mathbb{C}G$-modules when $G = D_{10}$.

4. Let $V_1, \ldots, V_k$ be a complete set of non-isomorphic irreducible $\mathbb{C}G$-modules, and let V and W be arbitrary $\mathbb{C}G$-modules. For $1 \leq i \leq k$ define
$$d_i = \dim(\mathrm{Hom}_{\mathbb{C}\mathrm{G}}(V, V_i)$$
and
$$e_i = \dim(\mathrm{Hom}_{\mathbb{C}\mathrm{G}}(W, V_i)).$$
Prove that
$$\dim(\mathrm{Hom}_{\mathbb{C}\mathrm{G}}(V, W)) = \sum_{i=1}^{k} d_i e_i.$$