MATH 473
FALL 2019
HOMEWORK 15

1. Find the irreducible characters of C_2 (the cyclic group with two elements).

2. Let $G = C_2 = \langle a : a^2 = e \rangle$, and let $\rho : G \to \mathrm{GL}(3, \mathbb{C})$ be the representation defined by
$$a\rho = \begin{bmatrix} 3 & 0 & -2 \\ 0 & 1 & 0 \\ 4 & 0 & -3 \end{bmatrix}.$$
Let V be $\mathbb{C}^3$, and make V into a $\mathbb{C}G$-module by defining $vg = v \cdot (g\rho)$ for $g \in G$. Find the character of V and write it as a sum of irreducible characters.

3. Determine the irreducible characters of the quaternion group Q.

4. Let ρ be a representation of the group G over $\mathbb{C}$.
   (a) Show that $\delta : G \to \mathrm{GL}(1, \mathbb{C})$ given by $\delta(g) = \det(g\rho)$ is a character.
   (b) Prove that $G/\operatorname{Ker}\delta$ is abelian.
   (c) Suppose that there is some $g \in G$ such that $\delta(g) = -1$. Prove that G has a subgroup of index 2.