MATH 473
FALL 2019
HOMEWORK 16

1. Recall that A_4 is the group of even permutations of S_4.
   (a) Prove that A_4 is generated by $(1,2,3)$ and $(1,4,2)$.
   (b) Prove that $(1,2,3)$ and $(1,4,2)$ are conjugate to each other in A_4.
   (c) Determine all the one-dimensional characters of A_4.

2. Determine all the irreducible characters of A_4.
   (Hint: You already have the one-dimensional ones. How many higher-dimensional ones are there, and what are their dimensions? Use the permutation module of A_4 to find a higher-dimensional character, and then prove that it must be irreducible.)

3. Let G be a group of order $2k$ where k is an odd integer. Using the regular representation of G, show that G has a normal subgroup of order k.

4. Prove that if $g \in G$ is a non-identity element, then there is some irreducible character χ of G such that $\chi(g) \neq \chi(1)$.