MATH 473 FALL 2019 HOMEWORK 18

1. Let $G = \langle a : a^3 = e \rangle$ be the cyclic group of order 3, $C_1 = \{e\}$, $C_2 = \{a\}$, and $C_3 = \{a^2\}$ its conjugacy classes. Denote by $K_i : G \to \mathbb{C}$ the characteristic function of C_i ; i.e., the function that is 1 on C_i and 0 on the rest of G.

Write the three characteristic functions K_1 , K_2 , K_3 as linear combinations of the irreducible characters of G.

- 2. Let $G = S_3$. Let $f : G \to \mathbb{C}$ be defined by f(e) = 11, f(g) = 3 for any two-cycle g, and f(h) = 5 for any three-cycle h. Use inner products to write f as a linear combination of the irreducible characters of G.
- 3. Let $G = \langle (1,2), (3,4) \rangle \subset S_4$. Decompose the permutation module for G into a direct sum of irreducible submodules. (Hint: Find the four irreducible characters of G, and use Proposition 14.26.)
- 4. Suppose that χ is a character of G and that for all $g \in G$, $\chi(g)$ is an even integer. Prove or disprove: $\chi = 2\psi$ for some character ψ of G.