MATH 473
FALL 2019
HOMEWORK 20

1. Let G be a group with k conjugacy classes represented by $g_1, \ldots, g_k$, and irreducible characters $\chi_1, \ldots, \chi_k$. Consider the character table of a group G as a square matrix A, with the row i column j entry given by $\chi_i(g_j)$. Let A^* denote the conjugate transpose of A, i.e. $A^* = \overline{A^T}$ this is often called the *adjoint* of A). Determine the matrix

$$A^*A.$$

2. (a) Let G be the group S_3. Determine $|\det(A)|$ for the matrix A described in problem 1.
   (b) Let G be an abelian group of order n. Determine $|\det(A)|$ for the matrix A described in problem 1.

3. For characters χ and ψ of G, Define the function $\chi\psi : G \to \mathbb{C}$ by $\chi\psi(g) = \chi(g)\psi(g)$. Prove the following.
   (a) If $\chi(e) = 1$, then $\chi\psi$ is a character of G.
   (b) If $\chi(e) = 1$, then $\chi\psi$ is an irreducible character of G if and only if ψ is irreducible.

4. Suppose that $\psi = \overline{\chi}$, and $\chi(e) > 1$. Prove that $\chi\psi$ can not be an irreducible character of G. (Note: $\chi\psi$ is a class function, but we do **not** know that it is a character. We will prove later that $\chi\psi$ is always a character.)