MATH 473
FALL 2019
HOMEWORK 21

1. Let $G = D_8 = \langle a, b | a^4 = b^2 = e, b^{-1}ab = a^{-1} \rangle$. Let ρ_1 be the representation defined by
$$a\rho_1 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}, \text{ and } b\rho_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$
and let V be the $\mathbb{C}G$-module $\mathbb{C}^2$ with $vg = v(g\rho_1)$. Let ρ_2 be the representation defined by
$$a\rho_2 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \text{ and } b\rho_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$
and let W be the $\mathbb{C}G$-module $\mathbb{C}_2$ with $wg = w(g\rho_2)$. Let $\{v_1, v_2\}$ and $\{w_1, w_2\}$ be the standard bases of V and W, respectively. Determine the matrices for multiplication by a and b on $V \otimes W$ with respect to the basis $\{v_1 \otimes w_1, v_1 \otimes w_2, v_2 \otimes w_1, v_2 \otimes w_2\}$.

2. Determine the character of $V \otimes W$ in problem 1.

3. Let χ, ψ, and θ be characters of G. Prove that
$$\langle \chi\psi, \theta \rangle = \langle \chi, \bar{\psi}\theta \rangle.$$

4. Decompose the square of the irreducible two-dimensional character of S_3 as a sum of irreducible characters.