

MATH 473
FALL 2019
HOMEWORK 22

1. Prove that for $\alpha_1, \dots, \alpha_r \in \mathbb{C}$, we have

$$\det \begin{pmatrix} 1 & a_1 & a_1^2 & \dots & a_1^{r-1} \\ 1 & a_2 & a_2^2 & \dots & a_2^{r-1} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & a_r & a_r^2 & \dots & a_r^{r-1} \end{pmatrix} = \prod_{1 \leq i < j \leq r} (a_j - a_i).$$

Hint: Use induction. Prove that

$$\det \begin{pmatrix} 1 & a_1 & a_1^2 & \dots & a_1^{r-1} \\ 1 & a_2 & a_2^2 & \dots & a_2^{r-1} \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & a_r & a_r^2 & \dots & a_r^{r-1} \end{pmatrix} = \det \begin{pmatrix} 1 & a_1 & a_1^2 & \dots & a_1^{r-2} & f(a_1) \\ 1 & a_2 & a_2^2 & \dots & a_2^{r-2} & f(a_2) \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 1 & a_r & a_r^2 & \dots & a_r^{r-2} & f(a_r) \end{pmatrix},$$

for any monic degree $r - 1$ polynomial f . Then choose

$$f(x) = \prod_{i=1}^{r-1} (x - a_i).$$

2. Let $T : V \otimes V \rightarrow V \otimes V$ be defined as on page 196. Prove that for any $v, w \in V$,

$$(v \otimes w)T = w \otimes v.$$

3. Let χ be the $|\text{Fix}| - 1$ character of S_3 . Determine χ_S and χ_A . Is either of them irreducible?

4. Let χ be a degree one character. Determine χ_S . What can you say about χ_A ?