MATH 473
FALL 2019
HOMEWORK 24

1. Let $G = S_4$ and let H be the subgroup $\langle (1\ 2\ 3\ 4), (1\ 3) \rangle$ of G. Note that H is isomorphic to the dihedral group of order 8.

   For each irreducible character χ of G, express $\chi \downarrow H$ as a sum of irreducible characters of H.

2. Let G be a group, and let H be a subgroup of G. Assume that H is abelian. Prove that for every irreducible character χ of G,
$$\chi(1) \leq [G : H].$$

3. Let $G = D_{2n} = \langle a, b | a^n = b^2 = e, b^{-1}ab = a^{-1} \rangle$ be the dihedral group of order $2n$. Prove that every irreducible character of G has degree one or two.

4. Let $G = D_{2n} = \langle a, b | a^n = b^2 = e, b^{-1}ab = a^{-1} \rangle$ be the dihedral group of order $2n$. Let χ be an irreducible degree two character of G. Prove that for any integer m, if we let $g = a^m b \in G$, then $\chi(g) = 0$.