MATH 473
FALL 2019
HOMEWORK 25

1. Suppose that G is a group with a subgroup of index 3, and let χ be an irreducible character of G. Prove that
$$\langle \chi{\downarrow}H, \chi{\downarrow}H\rangle_H = 1, 2, \text{ or } 3.$$

2. Give an example of groups G and H and an irreducible character χ of G as in problem 1 such that
$$\langle \chi{\downarrow}H, \chi{\downarrow}H\rangle_H = 2$$

3. Give an example of groups G and H and an irreducible character χ as in problem 1 such that
$$\langle \chi{\downarrow}H, \chi{\downarrow}H\rangle_H = 3$$

4. It is known that the complete list of degrees of the irreducible characters of S_7 is
$$1, 1, 6, 6, 14, 14, 14, 14, 15, 15, 20, 21, 21, 35, 35.$$
Also, A_7 has nine conjugacy classes. Find the complete list of degrees of the irreducible characters of A_7.