MATH 473
FALL 2019
HOMEWORK 26

1. Let $G = H \times K$, where H and K are groups, and let χ be an irreducible character of G. Prove that there are irreducible characters ψ of H and ϕ of K so that $\chi{\downarrow}H = \phi(1)\psi$ and $\chi{\downarrow}K = \psi(1)\phi$.

2. Let G be any group of order pq, where p and q are primes and $p \leq q$. Prove that every irreducible character of G has degree less than or equal to p.

3. Let $\mathbb{F}_5$ be the finite field with five elements. Let
$$G = \left\{ \begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix} : a, b \in \mathbb{F}_5, a \neq 0 \right\}$$
with the group operation of matrix multiplication. Determine the character table of G.

4. Let G be a finite group, and denote the identity of G by e.
   (a) Let χ be an irreducible character of G, and let ψ be an arbitrary character of G. Then
   $$\langle \chi, \psi \rangle \leq \frac{\psi(e)}{\chi(e)}.$$
   (b) Let χ, ψ, θ be irreducible characters G. Prove that
   $$\langle \chi\psi, \theta \rangle \leq \theta(e).$$