MATH 473
FALL 2019
HOMEWORK 27

1. Let $G = S_3$, and set $a = (1\ 2\ 3)$ and $b = (1\ 2)$. Define the following vectors in $\mathbb{C}G$ (where $\omega = e^{2\pi i/3}$):
$$u_1 = e + \omega a + \omega^2 a^2,$$
$$u_2 = b + \omega ab + \omega^2 a^2 b,$$
$$v_1 = b + \omega^2 ab + \omega a^2 b,$$
$$v_2 = e + \omega^2 a + \omega a^2.$$
Define $U = \text{span}(u_1, u_2) \subset \mathbb{C}G$. Prove that U is a $\mathbb{C}G$-submodule of $\mathbb{C}G$.

2. With definitions from problem 1, define
$$\theta : U \to \mathbb{C}G$$
by $u_1\theta = v_1$ and $u_2\theta = v_2$ and extending linearly. Prove that θ is a $\mathbb{C}G$-homomorphism, and find $r \in \mathbb{C}G$ such that $u\theta = ru$ for all $u \in U$.

3. Let $G = C_6 = \langle a | a^6 = e\rangle$. Let $H = \langle a^3 \rangle \subset G$. Let $U = \text{span}(1 + a^3) \in \mathbb{C}G$. Determine $U \uparrow G$ and decompose $U \uparrow G$ as a direct sum of irreducible $\mathbb{C}G$-modules.

4. Let $G = S_3$, and let $H = \langle (1\ 2\ 3)\rangle$. Let $U = \text{span}(\sum_{h\in h} h)$ be a $\mathbb{C}H$-submodule of $\mathbb{C}H$. Determine $U \uparrow G$, and decompose $U \uparrow G$ into a direct sum of irreducible $\mathbb{C}G$-modules.