MATH 473 FALL 2019 HOMEWORK 28

1. Let H be a subgroup of G. Prove directly from the definition, that for a $\mathbb{C}H$ -submodule U of $\mathbb{C}H$,

$$\dim(U \uparrow G) = [G:H] \dim(U).$$

- 2. Let H be a subgroup of G. Prove that for any character χ of H, $(\chi \uparrow G)(e) = [G:H]\chi(e).$
- 3. Let $G = S_4$ and let $H = \langle (1 \ 2 \ 3 \ 4), (1 \ 3) \rangle \cong D_8$. For each irreducible character ψ of H, write $\psi \uparrow G$ as a sum of irreducible characters of G.
- 4. Let H be a subgroup of G, let ψ be a character of H, and let χ be a character of G. Prove that

$$(\psi(\chi \downarrow H)) \uparrow G = (\psi \uparrow G)\chi.$$

Hint: Write the left side as $\sum d_i \chi_i$, where the χ_i are the irreducible characters of G. Compute d_i by an inner product, and then manipulate the expression using Frobenius reciprocity (twice) to show that the right hand side is given by the same sum.