

MATH 473
FALL 2019
HOMEWORK 29

1. Suppose that H is a subgroup of G , and let χ_1, \dots, χ_k be the irreducible characters of G . Let ψ be an irreducible character of H . Then for some integers d_i , we have

$$\psi \uparrow G = d_1 \chi_1 + \dots + d_k \chi_k.$$

Prove that

$$\sum_{i=1}^k d_i^2 \leq [G : H].$$

2. Let $G = S_4$, and let $H = \langle (1\ 2) \rangle \subset S_4$. Let ψ be the trivial character of H . Compute the values of the character $\psi \uparrow G$.
3. Let G be a finite group, and let $H = \{e\} \subset G$. Let ψ be the trivial character of H (indeed the only character of H). Compute the values of the character $\psi \uparrow G$, and write it as a linear combination of the irreducible characters of G .
4. Let H be a subgroup of G with $[G : H] = 2$, and let ψ be a character of H . Prove that if $\psi \uparrow G$ is irreducible, then there is exactly one character $\phi \neq \psi$ of H such that $\phi \uparrow G = \psi \uparrow G$.