MATH 473
FALL 2019
HOMEWORK 3

1. Let V be a vector space and let ϑ be an endomorphism of V. Show that ϑ is a projection if and only if there is a basis $\mathscr{B}$ of V such that $[\vartheta]_{\mathscr{B}}$ is a diagonal matrix with all diagonal entries equal to 1 or 0.

2. Let $V = \mathbb{R}^3$, and let $U = \langle (1, 2, 3), (1, 1, 1) \rangle$ be a subspace of V. Determine a subspace W of V such that $V = U \oplus W$.

3. let $V = \mathbb{R}^3$, let $U = \langle (1, 1, 1), (1, 1, 0) \rangle$ and let $W = \langle (1, 0, 0) \rangle$. Let $\mathscr{B} = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$ be the standard basis of W. Determine the matrix $[\pi]_{\mathscr{B}}$ of the projection map $\pi : V \to V$ that has U as its image and W as its kernel.

4. Prove or disprove: Let V be a finite dimensional vector space over F, and let U be a subspace. Let $u \in U$, and $v \in V - U$. Then there is a projection $\pi : V \to V$ with $\operatorname{Im} \pi = U$ such that $v\pi = u$.