## MATH 473 FALL 2019 HOMEWORK 35

1. The character table of  $S_5$  is:

	e	$(1\ 2)$	$(1\ 2)(3\ 4)$	$(1\ 2\ 3)$	$(1\ 2\ 3\ 4)$	$(1\ 2\ 3\ 4\ 5)$	$(1\ 2)(3\ 4\ 5)$
$ x^G $	1	10	15	20	30	24	20
$\chi_1$	1	1	1	1	1	1	1
$\chi_2$	1	-1	1	1	-1	1	-1
$\chi_3$	4	2	0	1	0	-1	-1
$\chi_4$	4	-2	0	1	0	-1	1
$\chi_5$	5	1	1	-1	-1	0	1
$\chi_6$	5	-1	1	-1	1	0	-1
$\chi_7$	6	0	-2	0	0	1	0

For each irreducible character  $\chi_i$  of  $S_5$ , determine  $\iota \chi_i$ .

2. Let  $G = S_5$ . Use corollary 23.17 to determine

$$|\{y \in G : y^2 = (1 \ 2 \ 3)\}|$$

Confirm your answer by listing the elements whose square is  $(1\ 2\ 3)$ .

- 3. Prove that every irreducible character of every group  $D_{2n}$  can be realized over the real numbers. (The character tables of these groups are given in chapter 18).
- 4. Let  $\rho$  be an irreducible representation of degree 2 of a group G and let  $\chi$  be the character of  $\rho$ . Prove that  $\chi_A(g) = \det(g\rho)$  for all  $g \in G$ . Prove also that  $\iota \chi = -1$  if and only if  $\det(g\rho) = 1$  for all  $g \in G$ .