

SUPPLEMENTAL HOMEWORK PROBLEMS

5A: Fix a prime p and let

$$G = \mathbb{Z}_{p^{e_1}} \times \mathbb{Z}_{p^{e_2}} \times \cdots \times \mathbb{Z}_{p^{e_k}},$$

where each $e_i \geq 1$, and $e_1 \geq e_2 \geq \cdots \geq e_k$. Define

$$f(e) = |\{i : e = e_i\}|$$

for every natural number e , and define

$$g(e) = \log_p(|\{x \in G : p^e x = 0\}|).$$

(Note that we are writing G additively.) Clearly $g(e)$ is determined by the isomorphism class of G . Determine $f(e)$ in terms of $g(e)$, thereby proving that the e_i are uniquely determined by the isomorphism class of G .