

### SUPPLEMENTAL HOMEWORK PROBLEMS

**5B:** Let  $G$  be the subgroup of  $GL_3(\mathbb{F}_2)$ , consisting of matrices whose third row is  $(0, 0, 1)$ . Let

$$H = \left\{ \begin{bmatrix} a & b & 0 \\ c & d & 0 \\ 0 & 0 & 1 \end{bmatrix} \in GL_3(\mathbb{F}_2) \right\}$$

and let

$$N = \left\{ \begin{bmatrix} 1 & 0 & e \\ 0 & 1 & f \\ 0 & 0 & 1 \end{bmatrix} \in GL_3(\mathbb{F}_2) \right\}.$$

Prove that  $N, H$  are subgroups of  $G$ , and that  $N \triangleleft G$ . Show that  $G$  is the semidirect product of  $N$  and  $H$  (for some  $\varphi : H \rightarrow \text{Aut}(N)$ ). Prove that  $G$  is isomorphic to  $S_4$ .