SUPPLEMENTAL HOMEWORK PROBLEMS

7A: Let R be an integral domain with quotient field F. If T is an integral domain with $R \subset T \subset F$, prove that F is isomorphic to the quotient field of T.

7B: Let R be an integral domain, and for each maximal ideal M, consider R_M as a subring of the field of fractions of R. Prove that

$$\bigcap_{M \text{ maximal}} R_M = R.$$

7C: Let $R = \mathbb{Z}_6$ and $S = \{\bar{2}, \bar{4}\} \subset R$. Prove that $S^{-1}R$ is a field with three elements.

7D: Prove that a commutative ring with $1 \neq 0$ is local if and only if for all $r, s \in R$, $r + s = 1$ implies that r or s is a unit.