SUPPLEMENTAL HOMEWORK PROBLEMS

3B: If $N \lhd G$, $|N|$ is finite, $H < G$, $[G : H]$ is finite, and the GCD

$$(|N|, [G : H]) = 1,$$

then $N < H$.

3C: If $f : G \to H$ is a homomorphism, H is abelian, and N is a subgroup of G containing $\ker f$, then $N \lhd G$.

3D: (Algebra Qual, Jan. 2015, Problem 2) Let G be an abelian group. Set $K = \{a \in G : a^2 = e\}$ and let $H = \{x^2 : x \in G\}$. Prove that H is a subgroup of G, $K \lhd G$, and $G/K \cong H$.