Review Sheet for Applications of Integrals

Non-Calculus Metaformulas

- Area of Rectangle = Height × Width
- Volume of Slab or Generalized Cylinder = Area × Thickness
- Area of Circular Disk = $\pi \times (\text{Radius})^2$
- Area of Washer = $\pi \times [(\text{Outer Radius})^2 - (\text{Inner Radius})^2]$
- Volume of Cylindrical Shell $\approx 2\pi \times$ Radius × Height × Thickness
- (x, y)-Moment = ((x, y)-coordinate of Centroid) × Area
- Work = Force × Distance
- Spring Force = $-k \times$ Displacement from Natural Position
- Gravitational Force = Weight
- Gravitational Force = mg
- Gravitational Force = $\dfrac{Gm_1m_2}{r^2}$
- Force = Pressure × Area
- Pressure = Density × Depth
- Volume of Solid of Revolution = $2\pi \times$ Area of Generating Region× Distance of Centroid of Generating Region from Axis of Revolution

Calculus Metaformulas (accounting for du)

- Volume by Shells = Integral of $2\pi \times$ Radius × Height × Thickness
- Pumping Work = Integral of Cross-sectional Area × Thickness × Density× Distance Layer is Moved
- Hydrostatic Force = Integral of Width of Layer × Thickness × Density× Depth of Layer
- Moment = Integral of Moments of Slices

Calculus Metaformulas (not accounting for du)

- Area = Integral of height
- Volume = Integral of Cross-sectional area

- Volume by Washers = Integral of $\pi \times [(\text{Outer Radius})^2 - (\text{Inner Radius})^2]$
- Volume by Shells = Integral of $2\pi \times \text{Radius} \times \text{Height}$

Calculus Formulas

- $A = \int_a^b [f(x) - g(x)]\,dx$
- $V = \int_a^b A(x)\,dx$
- $V = \pi \int_a^b [(f(x))^2 - (g(x))^2]\,dx$
- $V = 2\pi \int_a^b x[f(x) - g(x)]\,dx$
- $\overline{x}A = \int_a^b x[f(x) - g(x)]\,dx$
- $\overline{y}A = \dfrac{1}{2}\displaystyle\int_a^b [(f(x))^2 - (g(x))^2]\,dx$
- $\overline{x} = \dfrac{\overline{x}A}{A}$
- $\overline{y} = \dfrac{\overline{y}A}{A}$
- $W = \int_a^b F(x)\,dx$
- $W = \int_a^b \sigma|x - c|A(x)\,dx$
- $W = \int_a^b \sigma|x - c|w(x)\,dx$

Things to Remember

- Distances, areas, and volumes are nonnegative.
- Draw pictures with representative line segments, and use these pictures.
- Variable of integration corresponds to thin direction of line segment.
- Choose a coordinate system and set up integrals consistently therewith.
- In formulas for volumes of solids of revolution, a radius is the distance of something from the axis of revolution.
- Horizontal and vertical distances are the absolute value of the difference of corresponding coordinates.
- Depth is distance below the surface of the liquid.
- The centroid of a region does not have to be within the region.
- No straight line can separate a region from its centroid.
- $a^2 - b^2 \neq (a - b)^2$