Logical Outline of Chapter 3

1. Dilation has the expected effect on the size of special polygons.

2. Dilation has the expected effect on the size of arbitrary sets.

3. Nonsingular matrix multiplication expands each open set by the same factor by which it expands $[0,1)^n$.

4. Nonsingular matrix multiplication expands each open set by the same factor.

5. Nonsingular matrix multiplication expands each set by the same factor.

6. There are sets of positive size that when multiplied by the elementary matrix E get expanded by the factor $|\det(E)|$.

7. Multiplication by the elementary matrix E expands arbitrary sets by the factor $|\det(E)|$.

8. Determinants are multiplicative.

9. Every nonsingular matrix is the product of elementary matrices.

10. Multiplication by the nonsingular matrix T expands arbitrary sets by the factor $|\det(T)|$.

11. If T is orthogonal, then $|\det(T)| = 1$.

12. Orthogonal matrix multiplication preserves size.

13. $\lambda(\{0\} \times \mathbb{R}^{n-1}) = 0$.

14. Proper (linear) subspaces of $\mathbb{R}^n$ have measure zero.

15. Singular matrix multiplication maps $\mathbb{R}^n$ to a proper linear subspace of $\mathbb{R}^n$.

16. Multiplication by the singular matrix T shrinks size to 0.

17. Multiplication by a matrix T expands arbitrary sets by the factor $|\det(T)|$.

18. Translation preserves size of special polygons.

19. Translation preserves size of arbitrary sets.

20. **Rigid motions preserve size.**