Math 214 Hwk 7

Problem 1. *Using the chain rule, find* $\partial w/\partial s$ *and* $\partial w/\partial t$*, where*

$$w = \sin(2x+3y), \quad x = s+t, \quad \text{and} \quad y = s-t.$$

Express your answer in terms of s *and* t*.*

Problem 2. *Show that*

$$u(x,t) = \frac{1}{2}[f(x-ct) + f(x+ct)]$$

is a solution to the one-dimensional wave equaton

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}.$$

Problem 3. *Show that any tangent plane to the cone* $z^2 = a^2x^2 + b^2y^2$ *passes through the origin.*

Problem 4. *A corporation manufactures a product at two locations. The cost of producing* x_1 *units at the first location is*

$$C_1 = 0.02x_1^2 + 4x_1 + 500.$$

The cost of producing x_2 *units at the second location is*

$$C_2 = 0.05x_2^2 + 4x_2 + 275.$$

If the product sells for \$15 *per unit, find the quantity that should be produced at each location to maximize profit.*

Problem 5. *Show that a triangle is equilateral if the product of the sines of its angles is maximum.*

Problem 6. *Use Lagrange multipliers to prove that the product of 3 positive numbers* x, y, z*, whose sum has the constant value* S*, is maximum when the 3 numbers are equal. Use this result to prove that*

$$\sqrt[3]{xyz} \le \frac{x+y+z}{3}, \quad x,y,z > 0.$$