Math 290 Homework 32.5, due Wednesday April 5

You may work together in groups of up to three students.

- (1) Write $[a]_n$ for the equivalence class of the integer a in the set \mathbb{Z}_n . Prove or disprove: The rule $f([x]_6) = [x]_3$ describes a well-defined function $f : \mathbb{Z}_6 \to \mathbb{Z}_3$. (2) Prove that the function $f : \mathbb{R} - \{3\} \to \mathbb{R} - \{6\}$ given by $f(x) = \frac{6x-5}{x-3}$ is bijective and
- find its inverse.
- (3) Let $f: A \to B$ and $g: B \to C$ be functions, and suppose that $g \circ f$ is bijective. Prove or disprove: f is injective.
- (4) Let $f: A \to B$ and $g: B \to C$ be functions, and suppose that $g \circ f$ is bijective. Prove or disprove: q is injective.
- (5) Prove that the set $\{\pi, e\} \times \mathbb{Q}$ is countably infinite.
- (6) Prove that the sets $A = [-2, -1] \subseteq \mathbb{R}$ and $B = [4, 5) \cup [2, 3] \cup \{6\} \subseteq \mathbb{R}$ have the same cardinality. (You may use the Schröder-Bernstein theorem.)