Math 290 Homework 37, due Wednesday April 19

You may work together in groups of up to three students. For T/F and multiple choice questions, explain your answers.

(1) True or False: If the series $\sum_{n=1}^{\infty} a_n$ does not converge, then $\lim_{n\to\infty} a_n \neq 0$.

(2) True or False: If $d > 0$, then an arithmetic sequence with first term c and common difference d will never converge.

(3) Which of the following is the negation of the statement

$$\text{“}\lim_{x\to a} f(x) = L\text{”?}$$

a) For all $\epsilon > 0$, there exists a $\delta > 0$ such that $0 < |x-a| < \delta$ implies $|f(x)-L| < \epsilon$.

b) There exists an $\epsilon \leq 0$ such that for all $\delta \leq 0$, there is an $x \in \mathbb{R}$ with $0 \geq |x-a| \geq \delta$ such that $|f(x) - L| \geq \epsilon$.

c) There exists an $\epsilon > 0$ such that for all $\delta > 0$, there is an $x \in \mathbb{R}$ with $0 < |x-a| < \delta$ such that $|f(x) - L| \geq \epsilon$.

d) For all $\epsilon > 0$, there exists a $\delta > 0$ such that $0 < |x-a| < \delta$ implies $|f(x)-L| \geq \epsilon$.

(4) In proving that $\lim_{n\to\infty} \frac{3}{n} = 0$, which of the following is the best choice for N, given an arbitrary $\varepsilon > 0$?

a) $N = 3\varepsilon$

b) $N = 3$

c) $N = 1/\varepsilon$

d) $N = 3/\varepsilon$

e) $N = 1/3\varepsilon$

(5) Which of the following statements is false?

a) A geometric sequence with common ratio r satisfying $|r| < 1$ converges to 0.

b) If a series $\sum_{n\in\mathbb{N}} a_n$ converges, then $\lim_{n\to\infty} a_n$ exists.

c) At the point $x = a$, if the function $f(x)$ has limit L and the function $g(x)$ has limit M, then the limit of the function $\left(\frac{f}{g}\right)(x)$ must be L/M.

d) All quadratic polynomials with real coefficients are continuous functions from $\mathbb{R}$ to $\mathbb{R}$.

e) None of these

(6) Using the $\varepsilon - \delta$ definition of a limit, prove that $\lim_{x\to 2} x^2 + 5x = 14$.

(7) Using the $\varepsilon - \delta$ definition of a limit, prove that $\lim_{x\to 1} \frac{2x+3}{3x+4} = \frac{5}{7}$.