Math 320 Final Exam Study Guide

1. General information

- This study guide mostly covers new material since the last midterm, but the final exam will cover all material we have done this semester. To review older material you should review the previous study guides, exams, and homework problems.
- The exam will be in class from 7-10 AM on Tuesday December 13.
- Books, notes, and calculators will not be allowed.
- The study guides are not guaranteed to be exhaustive. Not appearing on the study guides does not mean that something will not appear on the exam.

2. Definitions

Know the definitions discussed in the book and in class, including:

(1) Importance, inversion, rejection sampling
(2) Hash table
(3) L^2 inner product
(4) Fourier series of a function f
(5) Piecewise Lipschitz
(6) Discrete inner product
(7) Discrete Fourier transform
(8) Circular convolution
(9) Hadamard product
(10) Band limited, Nyquist frequency, Nyquist rate
(11) Alias
(12) Haar father function and mother wavelet

Be able to produce examples and non-examples for these definitions.

3. Theorems/algorithms you should know and be able to state and be able to use

Be sure to know the full statement of each theorem, including all hypotheses.

(1) Monte Carlo estimate for integrals (uniform and nonuniform distributions)
(2) Orthonormality of $\{e^{i\omega kt}\}$ (Theorem 8.2.1) (Which inner product are we using?)
(3) Convergence of Fourier series (Theorem 8.2.16)
(4) Fast Fourier transform (including Lemma 8.5.17)
(5) Finite convolution theorem
(6) Periodic sampling theorem
(7) Haar sons form an orthogonal basis for V_j and can be scaled to be orthonormal
(8) Haar daughter wavelets form an orthogonal basis for W_j, the orthogonal complement of V_j inside V_{j+1}
(9) The space V_{j+1} can be decomposed into $V_{j+1} = W_j \oplus_\perp W_{j-1} \oplus_\perp \ldots \oplus_\perp W_0 \oplus_\perp V_0$.
(10) Fast wavelet transform

4. Sample problems

(1) Prove a statement such as (2) above.
(2) In what settings would a wavelet decomposition be more useful than a Fourier series? Explain.
(3) Describe the FFT and explain why its temporal complexity is $O(n \log n)$.
(4) Compute the discrete Fourier transform of the vector $(2, -1, 4) \in \mathbb{C}^3$. Explain what this represents.
(5) Choose four constants a, b, c, d. Given a function f that is equal to a on $[0, 0.25)$, is equal to b on $[0.25, 0.5)$, is equal to c on $[0.5, 0.75)$, is equal to d on $[0.75, 1)$, and is periodic with period 1, define f at the points $0, 0.25, 0.5, 0.75$ appropriately and find the exponential Fourier series of f.
(6) Choose two vectors $\mathbf{a}, \mathbf{b}$ in $\mathbb{R}^4$. Compute the convolution $\mathbf{a} * \mathbf{b}$ directly. Compute the convolution using the DFT, Hadamard product, and inverse DFT instead.
(7) Given a distribution D with pdf equal to $f(x) = (1 + x)^{-2}$ on $[0, \infty)$, explain how to use a sample from the uniform distribution on $[0, 1)$ to obtain a sample from D.
(8) If $f_X(t)$ is the pdf for the standard normal distribution, write $\mathbb{E}\left[\frac{\mathbb{1}_{[1,1+\ln(2)]}(t)e^t}{f_X(t)}\right]$ as an integral and compute its value. If you sample from the normal distribution 10 times, how would you compute the standard error when using this sample to compute the value of the integral? Explain.