Homework 10, due October 9

(1) (Page 105, problem 10) A group of people are arranging themselves for a parade. If they line up three to a row, one person is left over. If they line up four to a row, two people are left over, and if they line up five to a row, three people are left over. What is the smallest possible number of people?
(2) Find the smallest nonnegative solution to the system of congruences

$$19x \equiv 103 \pmod{900},$$
$$10x \equiv 511 \pmod{841}.$$

(3) Use fast modular exponentiation to compute $3^{75} \pmod{103}$ by hand. Show each step as on page 79 of the text.
(4) Prove or provide a counterexample for each.
   (a) $\gcd(n, \varphi(n)) > 1$.
   (b) If $d|m$, then $\varphi(d)|\varphi(m)$.
   (c) If the same primes divide m and n, then $n\varphi(m) = m\varphi(n)$.