Homework 4, due September 15

(1) The Fibonacci numbers are given by $F_1 = 1, F_2 = 1, F_n = F_{n-1} + F_{n-2}$. True or false: Any two consecutive Fibonacci numbers are relatively prime. If true, prove it. If false, give a counterexample.
(2) Find all solutions modulo 17, 97, and 105, respectively, to the congruences $5x+3 \equiv 9 \pmod{17}$, $18y - 23 \equiv 5 \pmod{97}$, and $14x + 3 \equiv 10 \pmod{105}$. Show your work.
(3) Let a be a positive integer with $\gcd(a, 26) = 1$. Suppose $A = 0, B = 1, C = 2$, etc. Why will the function $x \mapsto ax^2 \pmod{26}$ not work as a cipher system? For example, if $a = 11$, then $C = 2 \mapsto 11 \cdot 2^2 \equiv 18 \pmod{26} = S$.
(4) For the following ciphertext, the first part was encrypted by a shift cipher. Decrypt both parts.

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DROXOHDMSZROBSCKPPSXODROPSBCDDGYVODDOBCKBOIY
VJLKNATDXKHTHAXAPIHTYTJHCHTNIRJTCFXKCNHAIVFJKKXFC
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