Homework 8, due September 29

(1) **New codebook**: ROT3. Let us invent our own codebook, called ROT3. The key k and input x are both 3-bit binary numbers. Define the output to be $ROT3(k, x) = k \oplus \text{RotateLeft}(x)$, where RotateLeft($x$) cyclically rotates by 1 bit to the left. Note that the plaintext gets rotated, not the key. For example, $ROT3(011, 100) = 011 \oplus \text{RotateLeft}(100) = 011 \oplus 001 = 010$.

Let's use the ROT3 codebook above. Suppose we use the key `101` and plaintext `000111101`. What is the ciphertext in each of the following modes?

(a) Electronic Codebook mode (ECB).
(b) Cipher Block Chaining mode (CBC) using an Initial Value of 110.
(c) Cipher Feedback mode using Initial Value 110. (O_j and X_j should be 3 bits long.)
(d) Output Feedback mode using Initial Value 110. (O_j and X_j should be 3 bits long.)
(e) Counter Mode (CTR) using Initial Value 110. (Note that $111 + 1 = 000$, as we are working with 3-bit binary numbers.)

(2) As in the previous problem, let us use the ROT3 codebook. Suppose we use the key `101` and ciphertext `101111000`. Decrypt to get the plaintext if the message was sent in the following mode. Draw diagrams to show your work.

(a) Electronic Codebook mode (ECB).
(b) Cipher Block Chaining mode (CBC) using an Initial Value of 110.
(c) Cipher Feedback mode using Initial Value 110.
(d) Output Feedback mode using Initial Value 110.
(e) Counter Mode (CTR) using Initial Value 110.

(3) (Page 110, problem 33) Show that the only irreducible polynomials in $\mathbb{Z}_2[X]$ of degree at most 2 are $X, X + 1$, and $X^2 + X + 1$. Show that $X^4 + X + 1$ is irreducible in $\mathbb{Z}_2[X]$.

(4) Show that $X^2 + 2$ is irreducible in $\mathbb{Z}_5[X]$. Find the multiplicative inverse of $1 + 2X$ in $\mathbb{Z}_5[X]$ (mod $X^2 + 2$).